

Multivariate Time Series Modelling of Malaria Incidence in Gombe, Nigeria

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Abstract

In this study, multivariate time series techniques were employed to model the reported cases of malaria among the residents in Gombe Local Government area of Gombe State, Nigeria. These data were extracts from the records of the Specialist Hospital in Gombe State on the reported cases of malaria from 2009 to 2013. For convenience, the entire residents in this locality were grouped into four comprising the male (adult), the female (adult), the pregnant women and the paediatrics groups. Multivariate time series model selection was performed with special consideration given to all the possible models' parameters. The results showed evidence of cointegration among the observed series on the four groups of residents in the study area. As a result, a Vector Error Correction (VEC) model of lag two with one co-integration vector was found to best fits the data. As part of checks for models' adequacy, the Portmanteau test for serial correlation showed no serial correlation among the residuals ($p = 0.9814$) while the Jaque-Bera test showed that the residuals are multivariate normally distributed ($p = 0.2232$). Further results showed that reported cases of malaria in Gombe State is generally erratic (non-stationary) over the period with male (adult) having the highest number of malaria cases in 2009 while the paediatrics (children) had the highest number of malaria cases in 2010, 2011, 2012 and 2013. Consequently, the proportion of deaths due to malaria disease was the highest among the children throughout the study periods as compared to other groups. Results from this work would provide useful guides to the stakeholders at providing necessary measures at arresting the high mortality rate due to malaria among the most vulnerable group (paediatrics) in Gombe State of Nigeria.

Keywords: Multivariate time series, Malaria cases, Cointegration, Vector error correction (VEC) model

1.0 Introduction

Malaria has been a long life-threatening parasitic disease transmitted by female anopheles mosquitoes. It threatens 3.3 billion people or half of the world population living in the world's poorest countries [1]. An estimated 219 million malaria cases were recorded in 2010 with nearly 660,000 deaths [2]. The burden of malaria disease is higher among the under-fives in the African region. About 81% of the cases and 91% of deaths occurred in Africa whereas, under-fives malaria deaths accounted for about 86% of all malaria deaths in Africa [1].

Malaria is the third leading cause of death for children of under five years worldwide, after pneumonia and diarrheal disease [1]. Thirty countries in Sub-Saharan Africa account for 90% of global malaria deaths [2]. Nigeria, Democratic Republic of Congo (DRC), Ethiopia, and Uganda account for nearly 50% of the global malaria deaths [1, 2]. Malaria is the second leading cause of death from infectious diseases in Africa after HIV/AIDS [1]. Almost 1 out of 5 deaths of under-five children in Africa is due to malaria [2]. In semi-arid and highland regions of Africa, occurrence of malaria cases is unpredictable while epidemic malaria is a common problem causing deaths annually [3]. However, the risks of morbidity and mortality associated with malaria, particularly in semi-arid and highland regions, vary spatially and temporally [4]. Most malaria infections, particularly in sub-Saharan Africa, are caused by *Plasmodium falciparum*. Malaria presents a major socio-economic challenge to African countries since it is the region most affected. This challenge cannot be allowed to go unnoticed since good health is not only a basic human need but also a fundamental human right and a prerequisite for economic growth [5].

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Hermenegilde et al. [6] fitted a Generalized Additive Model to malaria cases for Forecasting Malaria Cases in Bujumbura, the capital and largest city of Burundi. Eskindir and Bernt[7] employed ARIMA model to Predict Plasmodium falciparum Malaria Incidence in Chano Mille, South Ethiopia. They observed that there was higher malaria incidence among males and children of 5 to 14 years of age. Tarekegn et al. [8] forecasted malaria incidence from historical morbidity patterns in epidemic-prone areas of Ethiopia using ARIMA models. Laari [9] employed Bayesian geostatistical approach to model malaria epidemiology in the Amanse West District of Ghana.

In another related study, Li and Lee[10] developed an augmented common factor model for modelling mortality dynamics in multiple populations. Li and Hardy[11] considered four possible ways to generalize a single-population mortality model to one that fits two or more populations. Cairns et al.[12] introduced a general framework for producing consistent mortality forecasts for a pair of related populations. Dowd et al.[13] designed a gravity mortality model for two related but different sized populations. A similar model was also proposed by Jarner and Kryger [14]. Zhou et al.[15] introduced a two-population mortality model with transitory jump effects and applied it to pricing catastrophic mortality securitizations. All these models were constructed in such a way that forecasts of life expectancies in different populations do not diverge in the long run.

In this work, patterns of Malaria infection in Gombe local government area of Gombe State, Nigeria were studied. Appropriate multivariate time series models were fitted to forecast the future malaria cases among the four identified population groups in this community. The impact of different incidences of malaria cases among these population groups on mortality due to malaria was also investigated.

2.0 Material and Methods

The data analysed in this work were reported cases of malaria among the residents in Gombe State, Nigeria. These data were extracts from the records of the Specialist Hospital in Gombe State on the reported cases of malaria from 2009 to 2013. The entire residents in this study area were categorised into four distinct biological groups of male (adult), female (adult), pregnant women and the paediatrics (children).

Gombe State with its capital in Gombe, is one of the 36 states that make up the federation of Nigeria. It is situated in the North-Eastern part of Nigeria on the latitude $10^{\circ}15'N$ of the Equator and longitude $11^{\circ}17'E$ of the Greenwich Meridian (see Fig 1). The state has two distinct climates, the dry season (November – March) and the rainy season (April – October) with an average annual rainfall of 850mm [16].

Gombe state, with an area of about 20,265 km² has a population of around 2,353,000 people going by 2006 national population census (NPC, 2006). The state has eleven local government areas which includes Gombe local government as shown in Fig 2.



Fig 1: Map of Nigeria showing the inset map of Gombe State

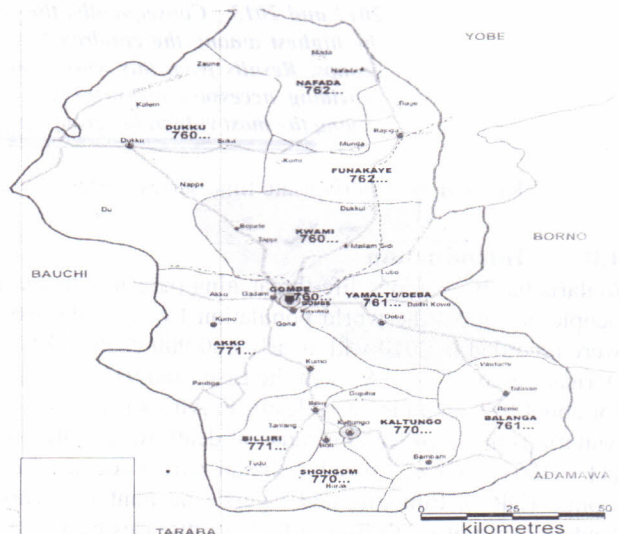


Fig 2: Map of Gombe State, Nigeria showing the ten local government areas(Source:[17])

2.1 Methodology

2.1.1 Vector Autoregressive (VAR) Models

The VAR model is one of the most successful, flexible, and easy to use models for the analysis of multivariate time series. It is a natural extension of the univariate autoregressive model to dynamic multivariate time series. The VAR model has proven to be especially useful for describing the dynamic behaviour of economic, financial and epidemiologic and health time series data and for forecasting[18]. Forecasts from VAR models are quite flexible because they can be made conditional on the potential future paths of specified variables in the model.

2.1.2 Stationary Vector Autoregressive Model

Let $Y_t = (y_{1t}, y_{2t}, \dots, y_{kt})'$ denote a $(k \times 1)$ vector of time series variables. The basic p -lag vector autoregressive (VAR(p)) model has the form [18].

$$Y_t = C + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \dots + \Pi_p Y_{t-p} + \varepsilon_t \quad (1)$$

where C denotes an $k \times 1$ vector of constants and Π_j is an $k \times k$ matrix of autoregressive coefficients for $j = 1, 2, \dots, p$. The $k \times 1$ vector ε_t is a vector generalization of white noise with

$$E(\varepsilon_t) = 0$$

and

$$E(\varepsilon_t, \varepsilon'_s) = \begin{cases} \Sigma, & \text{if } t = s \\ 0, & \text{if otherwise} \end{cases}$$

where Σ is a $(k \times k)$ symmetric positive definite variance-covariance matrix.

The starting point in Johansen's procedure in 1988 [19] in determining the number of co-integrating vectors is the VAR representation of Y_t . It assumed a vector autoregressive model of order p which is expressed as follows [20, 21]:

$$Y_t = \Lambda_1 Y_{t-1} + \Lambda_2 Y_{t-2} + \dots + \Lambda_p Y_{t-p} + \mu_t \quad (2)$$

where y_t is a k -vector of non-stationary $I(1)$ variables. If a non-stationary series, y_t must be differenced d times before it becomes stationary, then it is said to be integrated of order d . This would be written $y_t \sim I(d)$ and μ_t is a vector of innovations.

2.1.3 Vector Error Correction (VEC) Models

A vector error correction (VEC) model is a restricted VAR designed for use with non-stationary series that are known to be co-integrated. The VEC has co-integration relations built into the specification so that it restricts the long-run behaviour of the endogenous variables to converge to their co-integrating relationships while allowing for short-run adjustment dynamics. The co-integration term is known as the error correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments [20, 21].

When the variables are co-integrated, the corresponding error correction representations must be included in the system. By doing so, one can avoid misspecification and omission of the important constraints. Thus, the VAR in (2) can be re-parameterized as a Vector Error Correction Model (VECM) form [18, 22, 23] as

$$\Delta y_t = \alpha \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + \mu_t \quad (3)$$

with $\Pi_i = \alpha \beta' = -(I - \Lambda_1 - \dots - \Lambda_p)$ and $\Gamma_i = -(I - \Lambda_1 - \dots - \Lambda_i)$ $i = 1, \dots, p-1$

where the dimensions of α and β is $k \times r$ and r is the co-integration rank. The matrix α is the loading matrix and the coefficients of the long-run relationships are contained in β . The Γ_i matrix measures the transitory effects and μ_t is the vector of white noise.

3.0 Analysis and Results

The total reported cases of malaria from 2009 to 2013 as reflected by the data collected over the four groups of residents in Gombe are presented by the bar charts in Fig 3. From the bar charts, the male group had the highest number of malaria cases in 2009 while that of the Paediatric group was the highest in all the remaining years (2010 to 2013). The group of pregnant women recorded the least number of malaria cases over all the study periods.

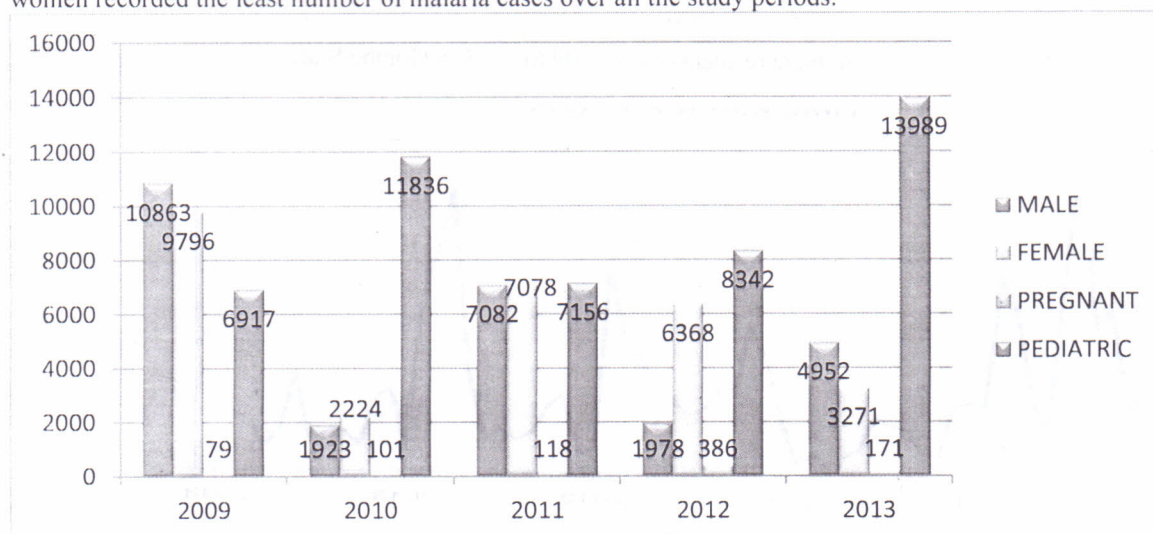


Fig 3: Bar-chart of total Malaria cases reported from 2009 to 2013 over the four groups of residents in Gombe Community, Nigeria.

However, the entire reported cases of malaria over the five years period is presented by the boxplot in Fig 4. From the box plot, it can be observed that the median case of malaria for the male group was the highest in 2009 followed by the female and then the paediatrics. Whereas, the median reported malaria cases was the highest for the paediatrics in 2010, 2012 and 2013.

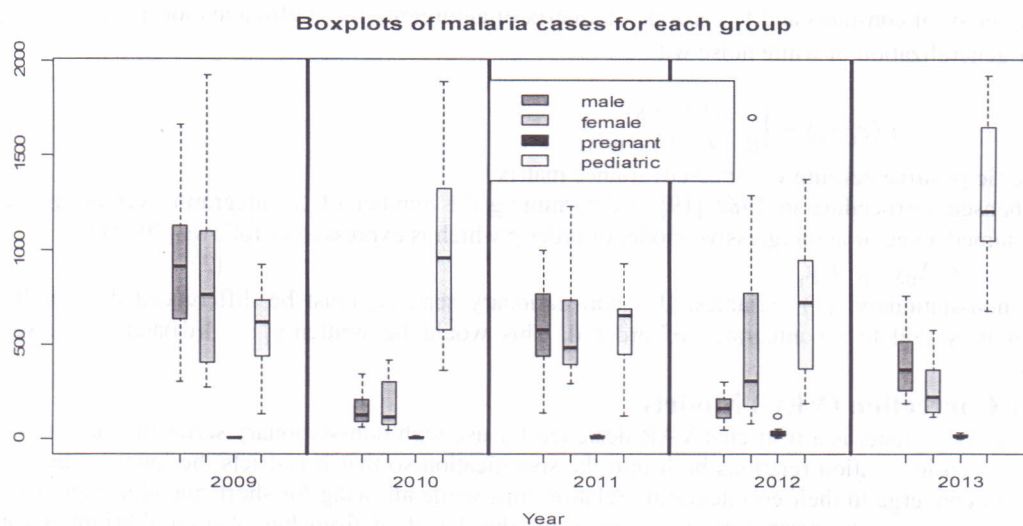


Fig 4: Boxplot of malaria cases reported from 2009 to 2013 over the four groups of residents in Gombe Community, Nigeria.

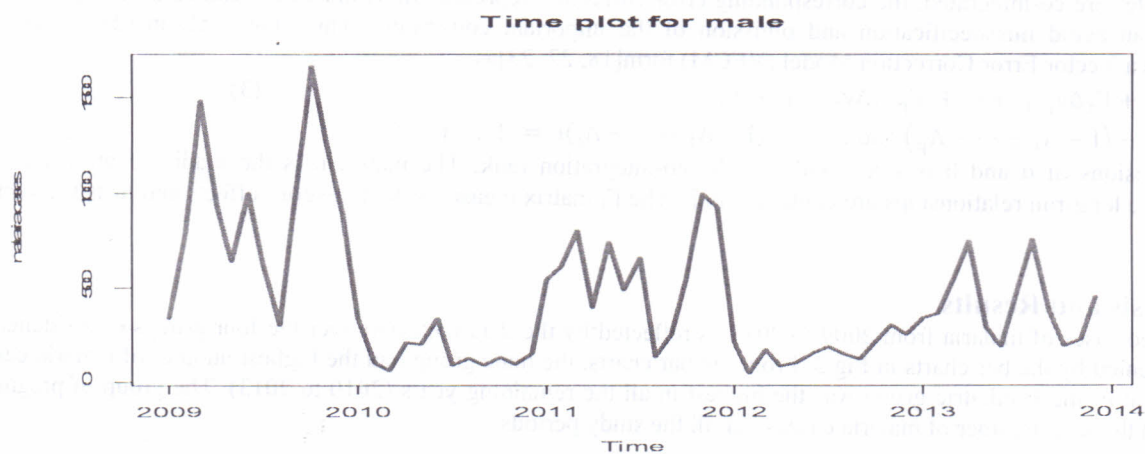


Fig 5: Timeplot of reported cases of malaria of male residents from 2009 to 2013 in Gombe State

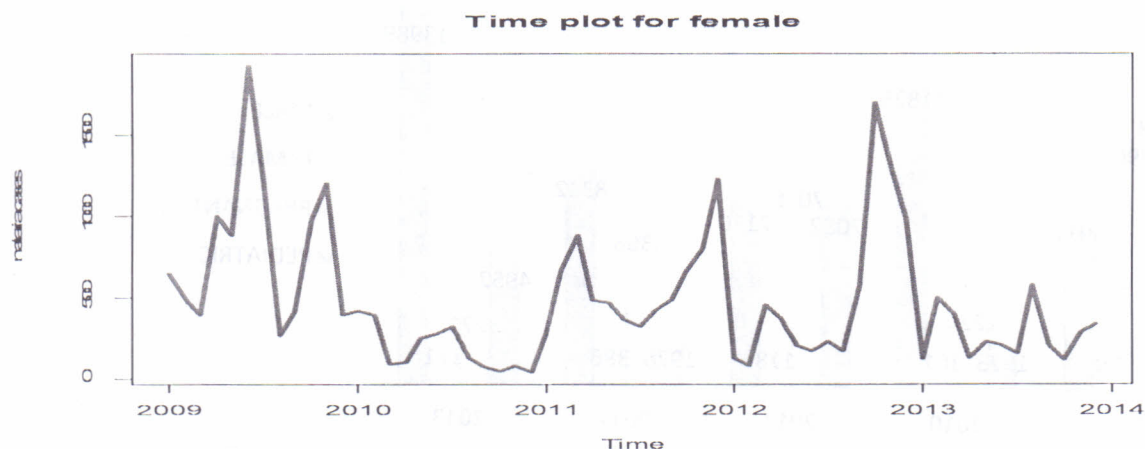


Fig 6: Timeplot of reported cases of malaria of female residents from 2009 to 2013 in Gombe State.

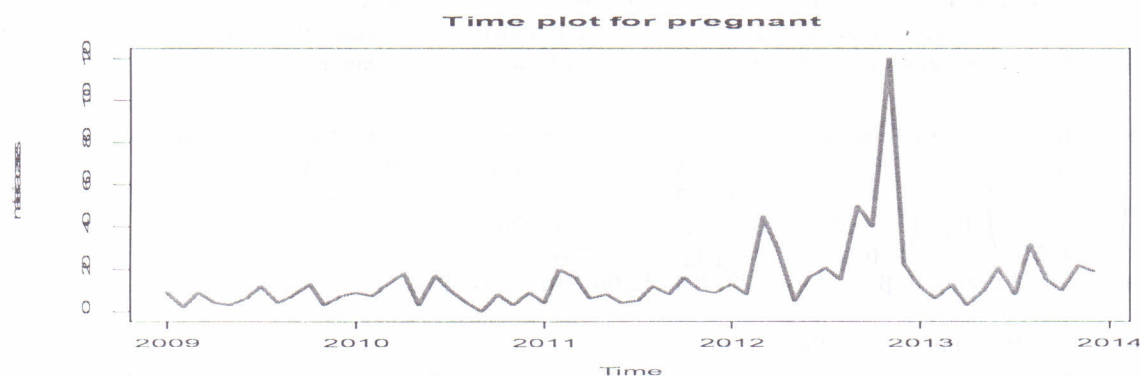


Fig 7: Timeplot of reported cases of malaria of pregnant women from 2009 to 2013 in Gombe State

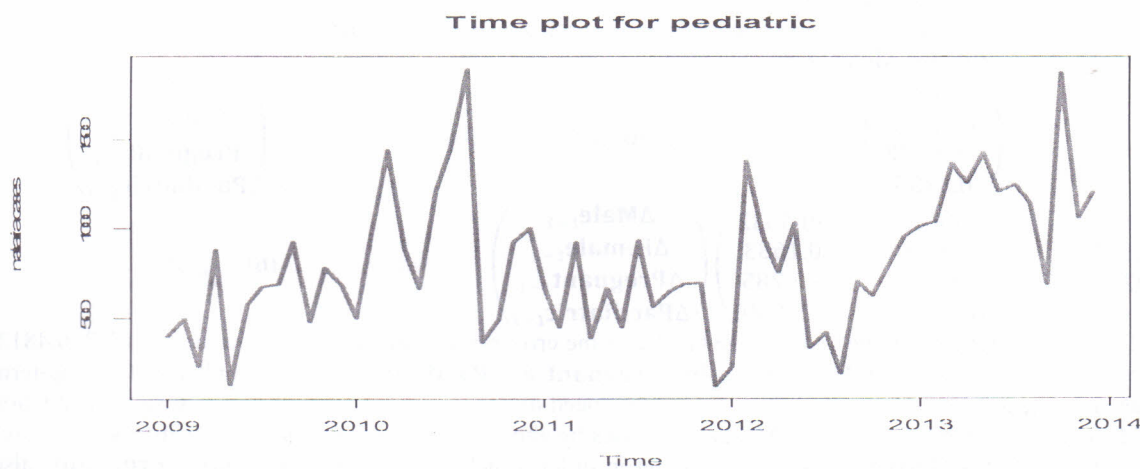


Fig 8: Timeplot of reported cases of malaria of paediatrics from 2009 to 2013 in Gombe State

In order to have clear overview of the structure of reported malaria cases in Gombe State, Nigeria, we present in Figs 5 to 8 the time plots of cases of malaria for the male, female, pregnant women and the paediatrics groups respectively.

Also, for easy assessment of the relative structures of the reported cases of malaria across the four population groups in Gombe, the time plots of malaria cases for the four groups are provided in Fig 9 from 2009 to 2013.

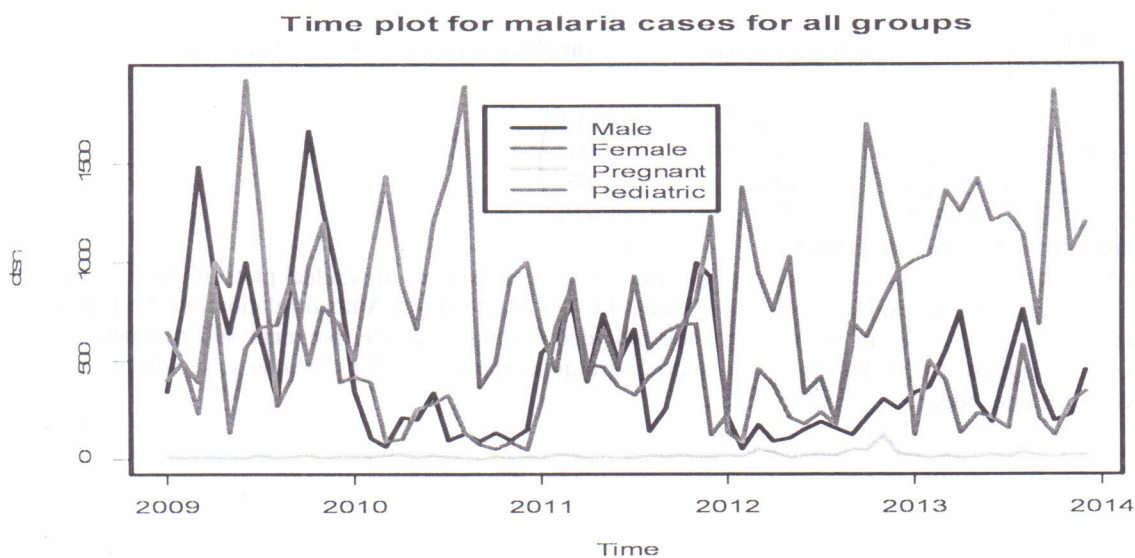


Fig 9: Timeplot of reported cases of malaria for the male, femal, pregnant women and the paediatrics in Gombe State from 2009 to 2013.

3.1 The Vector Error Correction Models (VECM) for Malaria Data

The VEC model of Lag $p = 2$ is fitted on the time series data on the reported malaria cases in Gombe State from 2009 to 2013. Recall that the co-integration rank r is 1. Thus, the appropriate VEC model for the data is

$$\Delta y_t = \alpha \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \mu_t \quad (4)$$

The β and α vectors and the Γ_1 matrix below were obtained using the `ca.jorls` function in package “`urca`” in R-package [24].

$$\beta = \begin{pmatrix} 1.0000 \\ -1.2701 \\ -0.0022 \\ -0.6194 \end{pmatrix}, \alpha = \begin{pmatrix} -0.1772 \\ 0.4812 \\ -0.0022 \\ 0.3155 \end{pmatrix}, \Gamma_1 = \begin{pmatrix} 0.2565 & 6.357 \times 10^{-2} & -3.248 \times 10^{-3} & -0.5532 \\ -0.2709 & 9.915 \times 10^{-2} & 1.535 \times 10^{-2} & 0.1683 \\ -1.688 & 6.892 \times 10^{-2} & -0.8702 & -2.785 \\ -0.1052 & 0.1057 & 7.656 \times 10^{-5} & -0.2626 \end{pmatrix}$$

Therefore, the co-integration vector is $\beta = (1, -1.2701404, 12.06193622, -0.6193633)$ and the equation of the long run equilibrium is

$$\text{Male} = 1.270 \text{Female} - 2.062 \text{Pregnant} + 0.619 \text{Paediatric} + 460.417 \quad (5)$$

The value 1.270 suggests that a unit increase in reported cases of malaria for Female induces, on the average, an increase of about 1.270 units in Male malaria cases. Similarly, unit increase in Pregnant cases leads to decrease of about 12.062 cases in Male group and a unit increase in Paediatric leads to increase of 0.619 cases in Male group.

Finally, using the error correction term as another independent variable in the unrestricted VAR model, the following estimates of the Vector Error Correction Model were obtained:

$$\begin{pmatrix} \Delta \text{Male}_t \\ \Delta \text{Female}_t \\ \Delta \text{Pregnant}_t \\ \Delta \text{Paediatric}_t \end{pmatrix} = \begin{pmatrix} -0.1772 \\ 0.4812 \\ -0.0022 \\ 0.3155 \end{pmatrix} \begin{pmatrix} 1, -1.2701, 12.0619, -0.6194 \end{pmatrix} \begin{pmatrix} \text{Male}_{t-1} \\ \text{Female}_{t-1} \\ \text{Pregnant}_{t-1} \\ \text{Paediatric}_{t-1} \end{pmatrix} + \begin{pmatrix} 0.2565 & 6.357 \times 10^{-2} & -3.248 \times 10^{-3} & -0.5532 \\ -0.2709 & 9.915 \times 10^{-2} & 1.535 \times 10^{-2} & 0.1683 \\ -1.688 & 6.892 \times 10^{-2} & -0.8702 & -2.785 \\ -0.1052 & 0.1057 & 7.656 \times 10^{-5} & -0.2626 \end{pmatrix} \begin{pmatrix} \Delta \text{Male}_{t-1} \\ \Delta \text{Female}_{t-1} \\ \Delta \text{Pregnant}_{t-1} \\ \Delta \text{Paediatric}_{t-1} \end{pmatrix} \quad (6)$$

Where: ‘ Δ ’ stands for first difference (D), the value in the bracket is the error correction term and the values -0.1772, 0.4812, -0.0022 and 0.3155 are the forces by which Male_t , Female_t , Pregnant_t and Paediatric_t are being pulled to the long-term equilibrium (i.e. they are adjustment coefficients that measured the speed of convergence to equilibrium). Results in equation (6) shows that for Male_t , the remaining long-term Male_t gap closes by about 17% in each period (the negative sign ensures that Female_t , Pregnant_t and Paediatric_t must be decreasing in order to achieve long run equilibrium), Pregnant_t also close 0.22% (the negative sign ensures that Male_t , Female_t , and Paediatric_t must be decreasing in order to achieve long run equilibrium), Female_t and Paediatric_t close by 48% and 32 % respectively.

β_{ij} Coefficients = $\begin{pmatrix} 1.0000 \\ -1.2701 \\ -0.0022 \\ -0.6194 \end{pmatrix}$ showed the Long-run (LR) equilibrium relationships between levels of variables.

α_{ij} Coefficients = $\begin{pmatrix} -0.1772 \\ 0.4812 \\ -0.0022 \\ 0.3155 \end{pmatrix}$ showed the amount of changes in the variables to bring the system back to equilibrium.

Γ_{ij} Coefficients = $\begin{pmatrix} 0.2565 & 6.357 \times 10^{-2} & -3.248 \times 10^{-3} & -0.5532 \\ -0.2709 & 9.915 \times 10^{-2} & 1.535 \times 10^{-2} & 0.1683 \\ -1.688 & 6.892 \times 10^{-2} & -0.8702 & -2.785 \\ -0.1052 & 0.1057 & 7.656 \times 10^{-5} & -0.2626 \end{pmatrix}$

3.2 Diagnostic Plots for the Residuals

The goodness of the fitted VEC model to the malaria data was examined through series of residual plots of the fitted models. In each of Figs 10, 11, 12, and 13, the residual plots of the fitted VEC models, the plots Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) of the residuals are provided for male, female, pregnant and paediatrics groups respectively. All the plots suggested that the fitted VEC models are quite appropriate for the time series data on reported cases of malaria in Gombe State, Nigeria.

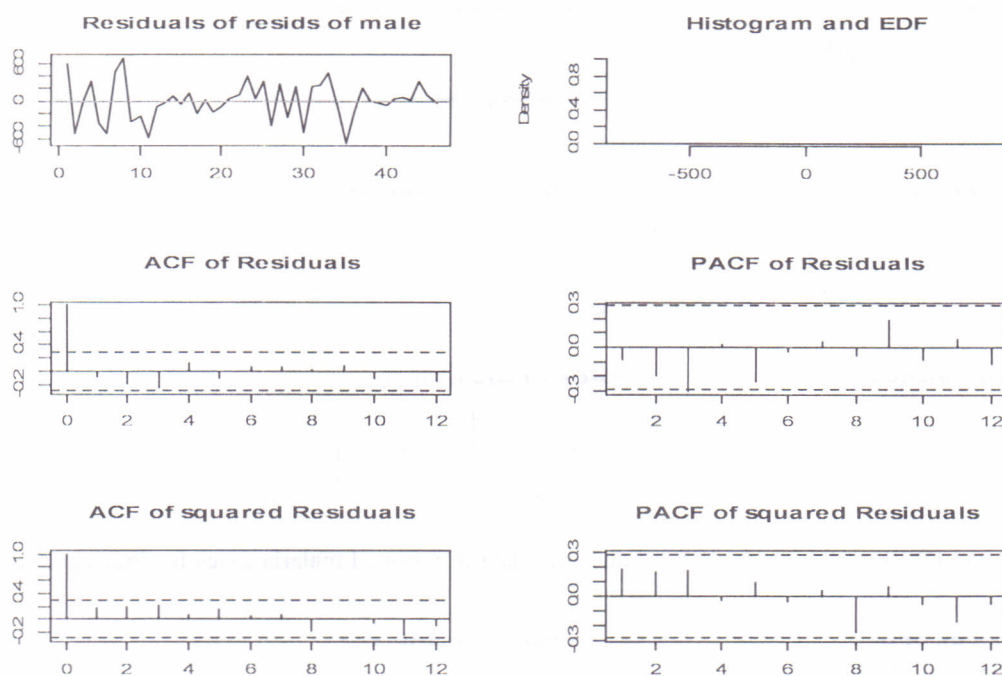


Fig 10: Residuals plot, ACF and PACF of the residuals for time series data of reported malaria cases for Male group.

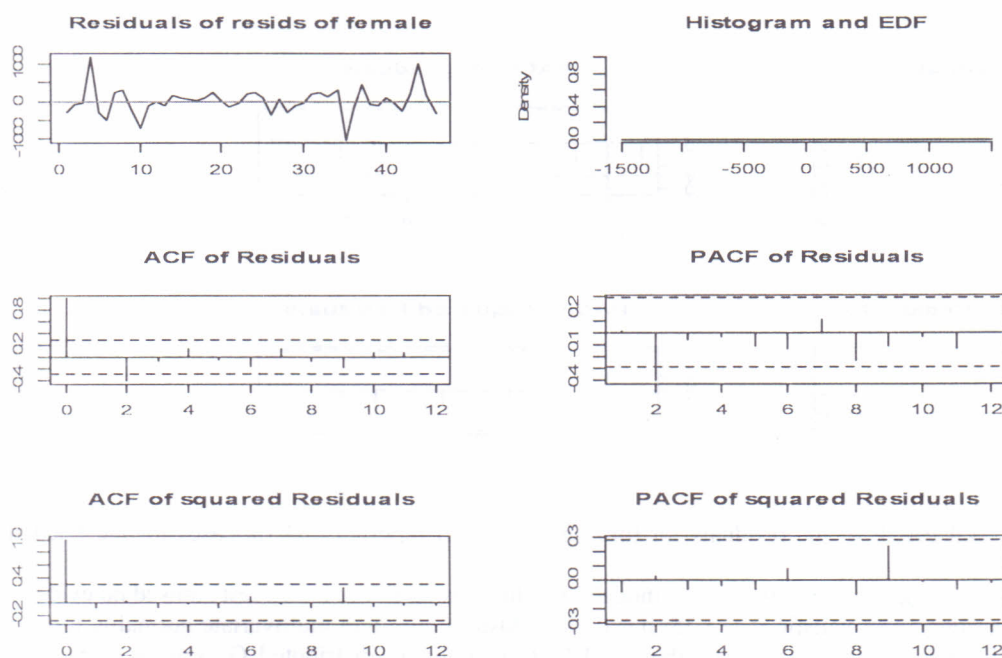


Fig 11: Residuals plot, ACF and PACF of the residuals for time series data of reported malaria cases for Female group.

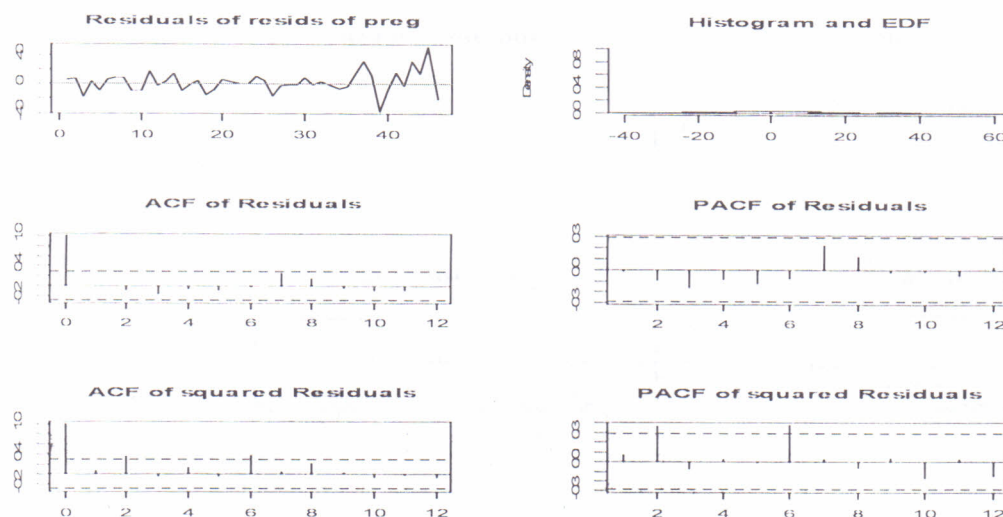


Fig 12: Residuals plot, ACF and PACF of the residuals for time series data of reported malaria cases for Pregnant women group.

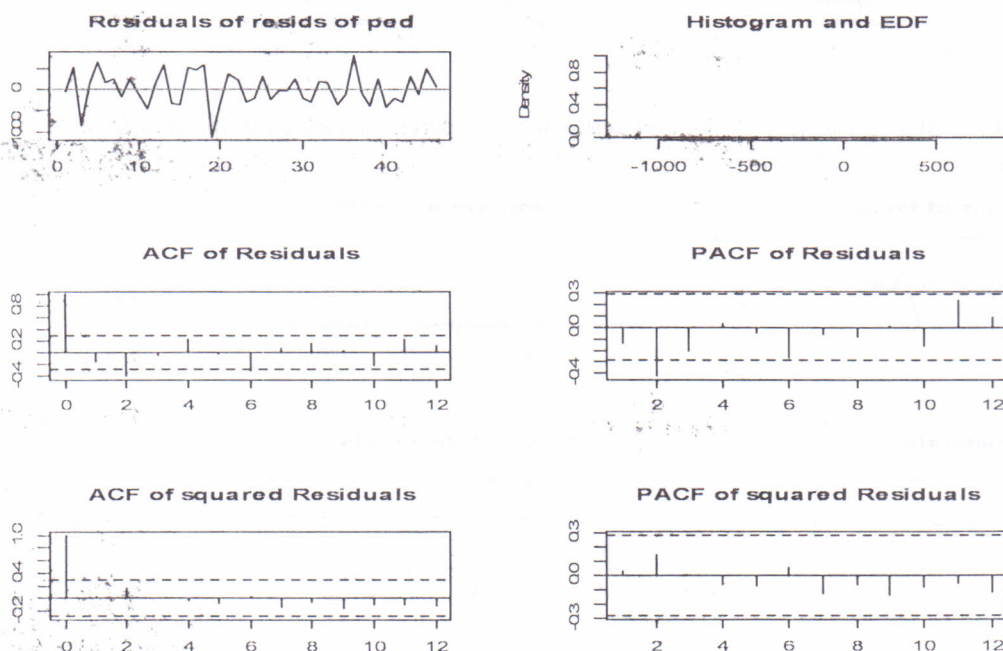


Fig 13: Residuals plot, ACF and PACF of the residuals for time series data of reported malaria cases for the Paediatrics group.

In addition, a test for the presence of serial correlation of the models' residuals using Portmanteau test showed no evidence of serial correlation among the residuals (χ^2 -statistic = 185.75; $p = 0.981$). Also, Jaque-Bera multivariate normality test on the residuals [25] indicated that the resulting residuals from the fitted VEC models are distributed Gaussian as expected (χ^2 -statistic = 5.69; $p = 0.223$).

4.0 Discussion and Conclusion

The pattern of malaria cases in Gombe State is generally erratic (non-stationary) over the periods considered (2009 to 2013). This might be as a result of some uncontrollable seasonal factors, such as climatic change and so on. Pattern of malaria cases within the male group increases beginning from 2009 through 2010 and later decreases afterwards.

The female group has its highest peak of malaria cases in 2009 and there was a decreasing trend beginning from 2010 towards 2011 with another spike towards the end of 2012.

There were apparently unnoticeable cases of malaria among the pregnant women in Gombe State between 2009 and 2012. However, the situation became pronounced throughout 2013.

The trend of malaria cases for the Paediatric group began just after 2009 with an increasing trend until it reaches its highest peak in June 2010. The trend exhibited a wavelike movement and went up in 2012 to the end of 2013.

The percentages of total cases recorded across the groups in Table A1 in the Appendix showed that, male(adult) had the highest number of cases in 2009. Paediatrics (children) recorded the highest number of malaria cases in 2010, 2011, 2012 and 2013. The level of infection of malaria by female(adult) was moderate throughout the study period while the pregnant women had the least number of cases. These results are clearly shown by the boxplot in Fig 4.

The percentages of total malaria cases recorded across the years in Table A2 in the Appendix showed that male (adult) and female (adult) had similar experience with highest number of cases in 2009 and least number of cases in 2010. The number of pregnant women infected by malaria was very high in 2012 and very small in 2010. Paediatric had highest number of cases in 2013 and the least number of cases in 2009.

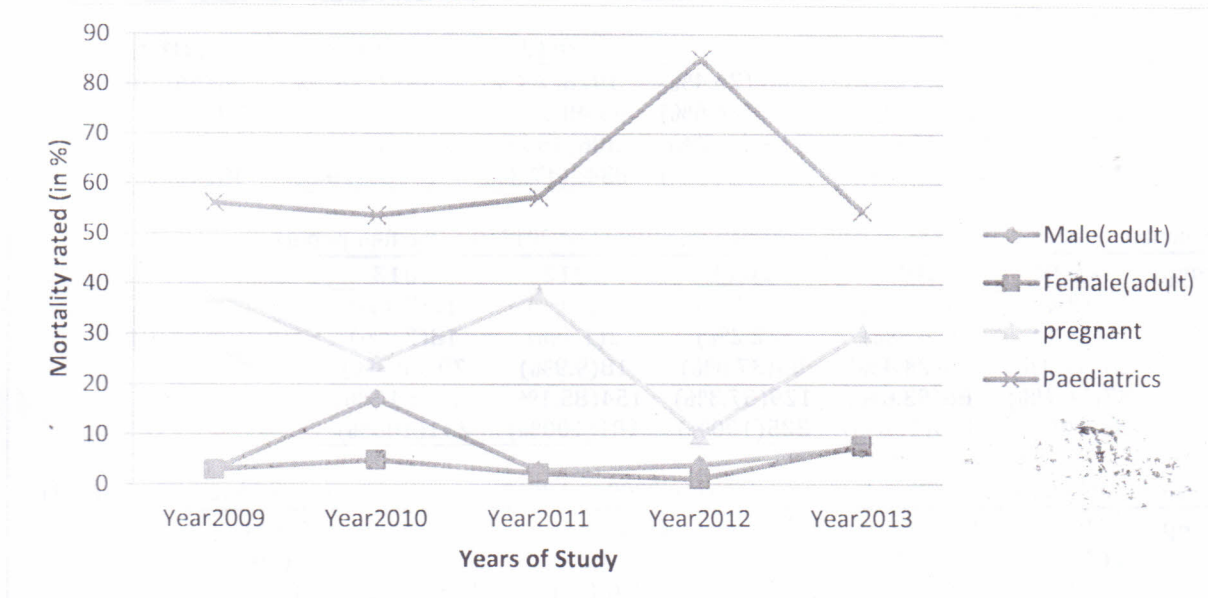


Fig 14: Line graphs of mortality experiences due to malaria infection across the four groups of residence in Gombe State over the study period (from 2009 to 2013).

As shown in Table A3 the percentages of mortality per 1000 malaria cases indicates that paediatrics (children) group had the highest mortality rates throughout the study period followed by the pregnant women. The child mortality rate due to malaria infection was above 50% of all the infected residence groups across the years of study with the worst child mortality rate of 85% recorded in 2012 as shown by the line graphs in Fig 14.

Results in Table A4 in the appendix showed that the mortality rate due to malaria infection among the male (adult) group was at the highest (39.6%) in 2010 with the least mortality rate within this group was 3.8% in 2009. Female (adult) group recorded the highest mortality rate of 54.5% in 2013 with the least mortality rate of 6.1% experienced in 2009 and 2012. The group of pregnant women recorded the highest mortality rate of 37.3% in 2011 and the least in 2012 (7.9%) and finally paediatrics had the highest and lowest mortality rates of 30.1% and 7.2% in 2012 and 2009 respectively.

In conclusion, it can be generally observed from the results in this work that the populations of paediatric (children) and pregnant women were the most vulnerable among the identified resident groups in Gombe State due to malaria infections. This confirmed an earlier results in the literature as reported by Sarki [26]. As a result, there is a need for federal government of Nigeria, the Gombe State government, the non-governmental organizations and all other stakeholders to develop appropriate measures to arrest the high spate of malaria infections and improve on malaria treatment and control, especially among the paediatrics and pregnant women in Gombe State. If necessary preventive measures are put in place, the high mortality rate among these most vulnerable resident groups would be drastically reduced.

Appendix

Table A1: Percentages of total cases of malaria recorded for the years 2009 to 2013 over the four groups considered.

Population Group	TOTAL CASES(%)				
	2009	2010	2011	2012	2013
Male	10863(39.3%)	1923(12.0%)	7082(33.0%)	1978(11.6%)	4952(22.1%)
Female	9796(35.4%)	2224(13.8%)	7078(33.0%)	6368(37.3%)	3271(14.6%)
Pregnant	79(0.3%)	101(0.6%)	118(0.6%)	386(2.3%)	171(0.8%)
Paediatrics	6917(25.0%)	11836(73.6%)	7156(33.4%)	8342(48.8%)	13989(62.5%)
Total	27655(100%)	16084(100%)	21434(100%)	17074(100%)	22383(100%)

Table A2: Percentages of total cases of malaria recorded for the years 2009 to 2013 over the years considered.

Population Group	TOTAL CASES(%)					
	2009	2010	2011	2012	2013	TOTAL
Male	10863(40.5%)	1923(7.2%)	7082(26.4%)	1978(7.4%)	4952(18.5%)	26798(100%)
Female	9796(34.1%)	2224(7.7%)	7078(24.6%)	6368(22.2%)	3271(11.4%)	28737(100%)
Pregnant	79(9.2%)	101(11.8%)	118(13.8%)	386(45.2%)	171(20.0%)	855(100%)
Paediatrics	6917(14.3%)	11836(24.5%)	7156(14.9%)	8342(17.3%)	13989(29.0%)	48240(100%)

Table A3: The number of deaths per 1000 malaria cases between 2009 and 2013 over the four population groups.

Population Group	2009	2010	2011	2012	2013
Male	2(3%)	21(17.1%)	6(2.7%)	7(3.9%)	17(7.4%)
Female	2(3%)	6(4.9%)	5(2.2%)	2(1.1%)	18(7.8%)
Pregnant	25(37.9%)	30(24.4%)	85(37.8%)	18(9.9%)	70(30.3%)
Paediatrics	37(56.1%)	66(53.6%)	129(57.3%)	154(85.1%)	126(54.5%)
Total	66(100%)	123(100%)	225(100%)	181(100%)	231(100%)

Table A4: The number of deaths per 1000 malaria cases between 2009 and 2013 within each year considered (2009 - 2013).

Population Group	2009	2010	2011	2012	2013	TOTAL
Male	2(3.8%)	21(39.6%)	6(11.3%)	7(13.2%)	17(32.1%)	53(100%)
Female	2(6.1%)	6(18.2%)	5(15.1%)	2(6.1%)	18(54.5%)	33(100%)
Pregnant	25(11%)	30(13.1%)	85(37.3%)	18(7.9%)	70(30.7%)	228(100%)
Paediatrics	37(7.2%)	66(12.9%)	129(25.2%)	154(30.1%)	126(24.6%)	512(100%)

5.0 References

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