Nig. J. Pure & Appl. Sci. Vol. 21 (2006):1988 – 1998
Printed in Nigeria
0794-0378
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COMPARISON OF TAGUCHI METHOD OF ANALYZING ROBUST PARAMETER DESIGN AND GRAPHICAL APPROACH

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ABSTRACT: Taguchi's method/approach of analyzing data from Robust Parameter Design (RPD) has received different criticisms, the most controversial being the choice of measure of performance, the Signal-To-Noise Ratio (SNR). The wrong choice of this measure of performance, SNR, can lead to cross-talk (confusion) of factors affecting Sensitivity (Mean) and those affecting Dispersion. An example of right choice of SNR is given and also when SNR is wrongly chosen. The graphical method through gamma-plot gives the appropriate choice of SNR and consequently gives a clear cut among factors affecting Sensitivity and those affecting Dispersion.

INTRODUCTION

Statistical Process Control (SPC) is used to reduce manufacturing imperfections in the product or process and keep the process in control. Though SPC is a powerful cost-saving and quality enhancing approach to reduce variability within the production phase, however SPC cannot compensate for poor quality design, hence the need to make use of OFF-LINE quality control principle. OFF-LINE quality control is principle of designing quality into a product at the design stage. If there is large variability in the performance of a product or process, prohibitively costly process control schemes may be required to improve the process capability and they cannot guarantee a product robust to variability. Apart from SPC cost, additional costs or expenses are also incurred due to service cost under warranty and most importantly loss of market share because of customers' dissatisfaction.

However, if the quality concept is moved down to the design process and product development stage (OFF-LINE Quality Control), then costly eventualities can be avoided. Therefore, the need for costly process control, mass inspection of products, and services costs are minimized. Building in quality at the design stage represents the latest phase in the evolution of quality system which is understood to have traversed four generally overlapping phases:

Inspection: this involves separation of bad from good through inspection. Though it has a valid and useful place in quality control system, its role is increasingly viewed as a tool for confirmation of good quality rather than for rectification of bad process.

Quality Control: it is a follow up to inspection. This is where the statistical methods are applied for first time to determine whether a process is in control on the basis of the findings of analysis of only a small sample (Shewart, 1938).

Quality Improvement: this is where the combination of the process control and data inspection is statistically analyzed to determine the source of problems. It is a constant effort towards manufacturing process improvement (Deming, 1982).

Quality by Design: this is designing quality into the process or product prior to the manufacturing the product. It incorporates the idea that
- A product can be made robust to the variation in the user's environment
- That the process, which produces the product, can be made robust to the normal variations in materials, components, and manufacturing before normal production begins (Peace, 1993).

ROBUST PARAMETER DESIGN

Taguchi (1987) pioneered Robust Parameter Design (RPD) as a strategy for improving the performance of a system, a process or a product. The concept rests on the importance of economically achieving high quality, low variability and constancy of functional performance of a product, a system or a process, using statistical methods, especially, statistically designed experiments, and concentrate on minimizing the deviation from target caused by uncontrollable factors/inputs, which he called NOISE FACTORS.

By making use of experimental design and statistical techniques, we can identify the appropriate settings of easy to control product (or process) parameter known as CONTROLLABLE INPUTS that reduce the sensitivity of engineering design to the source of UNCONTROLLABLE INPUTS. If a product can be designed so that it's output characteristics are resistance to all sources of uncontrollable factors, then it will function satisfactorily despite variability in its component parts and environmental conditions (Park, 1996). Therefore, reducing the effect of the uncontrollable factors rather than controlling them is a very cost effective way of improving engineering designs and developing stable and reliable products or manufacturing processes.

In RPD, a process can either be static or dynamic; hence we have a Static Parameter Design and a Dynamic parameter Design. A Static Parameter Design has a quality characteristic of interest that is at fixed level. A Dynamic Parameter Design has a quality characteristic that operates over a range of values and the goal is to improve the relationship between an input signal and an output response. Dynamic design is used to improve the functional relationship between input signal and output response. The output response should be directly proportional to the input signal (Fowlkes and Creveling, 1995). The ideal functional relationship between input signal and output response is a line through the origin.

An example of dynamic parameter design is an automotive acceleration experiment where the input signal is the amount of pressure on the gas pedal and the output response is the vehicle's speed. Let y_{ij} denote the observation corresponding to the ith setting of the control factors and jth setting of the signal factors, for $\bar{y}_i$ and $j = 1,2,\cdots,J$, under the assumption of a linear model,

$$\hat{y}_i = \alpha + \beta M + \varepsilon_{ij}$$

Where β_i is the sensitivity measure and var(ε_{ij})= σ_i^2, α is the intercept and it is zero if the line passes through the origin, while M is the signal inputs.

Taguchi Signal-To-Noise Ratio

Taguchi's original proposal for the analysis of quality improvement experiment was to classify the aim of the experiment as to maximize or minimize means as the case may be and at the same time minimize the variance. To achieve this, Taguchi proposes two-step procedures. The first step identifies control factors that are important for reducing variability and their appropriate settings are determined. In the second step, control factors that affect sensitivity are also identified together with their appropriate setting.

Considering Nominal-is-Best situation, factors that have greatest effect on variation are identified and the levels that minimize the variation are chosen. Once the variation is reduced, the remaining factors are possible candidates for adjusting the mean to target. A scaling factor is a factor in which its mean and standard deviation are proportional. A scaling factor has a significant effect on the mean with a relatively small effect on SNR. Thus one can use the scaling factor to adjust the mean on target and will not affect the SNR (Montgomery, 1996).

The basic performance measure proposed by Taguchi for studying variability is the signal-to-noise ratio (SNR).

$$SNR = -10Log\left(\frac{\beta_i^2}{\sigma_i^2}\right)$$ For Dynamic parameter design

$$SNR = -10Log\left(\frac{\mu_i^2}{\sigma_i^2}\right)$$ For Static parameter design

The SNR is calculated for each combination of factors used in the experiment. The standard methods of analysis such as ANOVA and normal probability plots can be used to judge the significant effects.

In communication, Signal-to-Noise Ratios (SNR) are commonly used as a measure of the goodness of transmission. SNR represents the strength (power) of the signal component divided by the strength of the noise (error) component. A parallel can be drawn with Taguchi's SNR by considering β as a signal and σ as noise. The SNR is based on the premise that by considering the underlying engineering of a system, it is usually possible to identify a factor that acts like amplification of the system. If this factor is used to adjust β, then there is a proportional change in σ, and thus β/σ stays constant.

The motivation behind Taguchi's Signal-to-Noise Ratio (SNR) is to uncouple location and dispersion effects. It can be shown that the use of SNR is equivalent to an analysis of the standard deviation of the logarithm of the original data. Thus, using SNR implies that a log transformation will always uncouple location and dispersion effects. Though, there is no assurance that this will always be the case (Phadke, 1989). A much safer approach is to investigate what type of transformation is appropriate.

Graphical Method

The graphical approach to the analysis is an extension of Taguchi's SNR method. The approach is devised by Lunani *et al* (1997). It is believed that the signal-to-noise adjustment factor has an effect on σ_i that is proportional to the effect on β_i (Box, 1988). If we consider a situation where the variance satisfies the following general relationship.

$$\sigma^2(c_i) = \beta^{\gamma}(c_i)\phi^2(c_i) \text{ for some } \gamma. \quad \ldots\ldots\ldots\ldots\ldots\ldots 1$$

Where c_i are the control variables/inputs $i = 1,2,\cdots I$.

ϕ is the dispersion parameter and β is the measure of sensitivity

then, $$\phi^2 = \frac{\sigma^2(c_i)}{\beta^{\gamma}(c_i)} \quad \ldots\ldots\ldots\ldots\ldots\ldots 2$$

This is reduced to Taguchi SNR when $\gamma = 2$.

However, there is no reason why γ should always equals to 2, hence one of the limitations of Taguchi SNR measure of performance. Therefore, equation 2 should be the appropriate measure of studying the dispersion. The next problem is how to determine γ. Taking the critical look at equation 2, there is log-linear relationship between the sensitivity measure β_i and the standard deviation σ_i.

$$Log\sigma_i(c_i) = Log\phi(c_i) + \frac{\gamma}{2}Log\beta_i(c_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

We can determine the nature of relationship between σ_i and β_i by plotting the estimated log-linear model in equation 3. If none of the control factors/inputs (c_i) has dispersion effects we have a very simple case, as $\phi(c_i)$ will be a constant. In such case, any systematic difference in standard deviation σ (c_i)'s are caused by the difference in the β (c_i)'s, therefore, the sensitivity–standard deviation plot will easily reveal the existence, if any, of the dependence between β_i and σ_i and the value of γ can then be estimated from the fitted line.

Estimation of γ through gamma-plot

The gamma-plot uses the principle of separation and parsimony to identify an appropriate value for γ. [Separation is the elimination of any unnecessary complications in the model due to functional dependence between mean and variance (or elimination of cross talk between location and dispersion effects) while Parsimony is the provision of simplest additive model. (Miller, 2003)] .

Considering the measure of dispersion in equation 3, fitting a series of models for the equation over a range of values for γ, the estimates of the factors effects are obtained for each γ (The estimated effects must have the same variance). If there are degrees of freedom to estimate the error, t-values for the factors effects are obtained and these t-values are plotted against various value of γ, and if there are no degrees of freedom to estimate the standard error, we use standardized contrast to estimate the effects. From the graph, the value of γ that differentiate dispersion and sensitivity factors is obtained. After identifying the factors that affect dispersion and those affecting sensitivity, the setting of these factors that minimize the dispersion and have the target at the desired point are then determined.

Examples

The first example is on AT&T layer growth experiment, which has 8 control factors and two noise factors (Location and Facet). The goal is to achieve a uniform thickness of about 14.5μm over the noise factors. The L16 (2^8) Taguchi design is used in the experiment, and it is carried out by running the complete set of noise factor settings at each combination of control factor settings. The design as well as the response data is shown in Table 1.

The trial mean and the corresponding SNR are the last two columns in Table 1. An examination of the Mean and SNR half-normal plots in Figure 1(a) and 1(b) respectively reveals that only factor D is significant for the mean effect while factors A and H are significant for the SNR. The plots of main effects from Sensitivity (Mean) and SNR are shown in Figures 2.

Having identified factors that are significant to the dispersion and sensitivity (Mean) effects, these factors are used to model the dispersion effect and sensitivity effect respectively.

The models are;

$$\hat{\eta} = -1.822 + 0.619x_A - 0.982x_H$$

$$\hat{y} = 14.352 + 0.402x_D$$

From the first model, the dispersion model (or using Fig.2), choosing A at negative (-) and H at positive (+) will maximize the SNR which is one of our objectives and from the second model, sensitivity model, D at positive level will give an approximate required layer growth of 14.754μm.

Table 1. Control Array of Experiment of Layer Growth with estimated Sensitivity ($\overline{Y}$) and SNR

Control Factor								Noise Factor									
								L-Bottom				L-Top					
A	B	C	D	E	F	G	H	M_1	M_2	M_3	M_4	M_1	M_2	M_3	M_4	$\bar{y}_i$	SNR
-	-	-	+	-	-	-	-	14.29	14.19	14.27	14.19	15.32	15.43	15.27	15.41	14.79	6.41
-	-	-	+	+	+	+	+	14.80	14.72	14.70	14.76	14.93	14.89	14.92	15.13	14.86	9.28
-	-	+	-	-	-	+	+	13.88	13.92	13.85	14.08	14.01	13.94	14.21	14.08	14.00	9.48
-	-	+	-	+	+	-	-	13.40	13.48	13.59	13.52	14.24	14.26	14.39	14.37	13.91	6.89
-	+	-	-	-	+	-	+	14.17	14.03	14.14	14.08	14.15	14.17	14.15	14.28	14.15	10.60
-	+	-	-	+	-	+	-	13.25	13.33	13.19	13.44	14.22	14.30	14.27	14.41	13.80	6.49
-	+	+	+	-	+	+	-	14.06	14.09	14.18	14.05	15.29	15.52	15.42	15.21	14.73	6.14
-	+	+	+	+	-	-	+	14.31	14.41	14.68	14.58	15.01	15.06	15.57	15.47	14.89	6.90
+	-	-	-	-	+	+	-	13.73	13.29	12.65	13.27	14.90	14.79	14.19	14.63	13.93	5.65
+	-	-	-	+	-	-	+	13.89	14.56	14.45	13.71	13.75	14.32	14.22	13.82	14.09	7.47
+	-	+	+	-	+	-	+	14.22	14.39	15.28	15.04	14.19	14.43	15.55	15.22	14.79	6.63
+	-	+	+	+	-	-	+	13.52	13.58	14.28	13.84	14.56	14.47	15.23	15.11	14.33	6.19
+	+	-	+	-	-	+	+	14.53	14.25	14.67	15.28	14.74	14.18	14.97	15.55	14.77	6.87
+	+	-	+	+	+	-	-	14.57	14.03	13.71	14.64	15.87	15.22	14.97	16.00	14.88	5.82
+	+	+	-	-	-	-	-	12.90	12.71	13.15	13.89	14.25	13.84	14.13	15.17	13.76	5.66
+	+	+	-	+	+	+	+	13.95	14.08	14.11	13.59	13.81	14.07	14.43	13.69	13.97	7.91

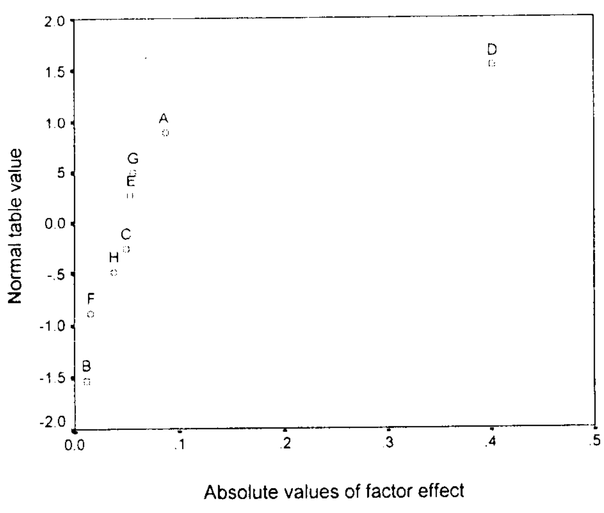

Fig. 1(a) Half-Normal plot for $\bar{y}$

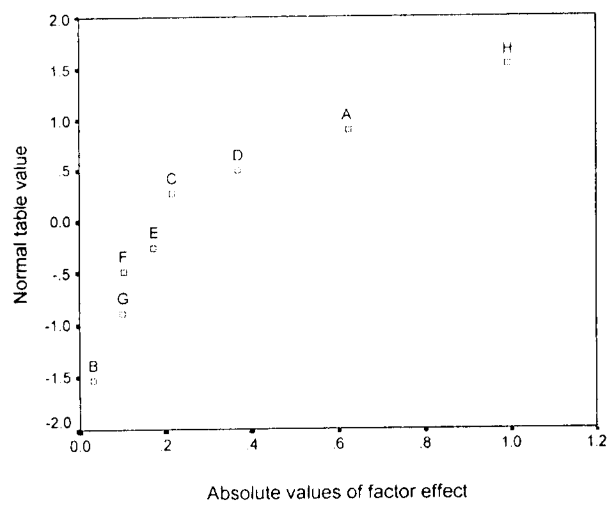

Fig. 1(b) Half-Normal plot for SNR

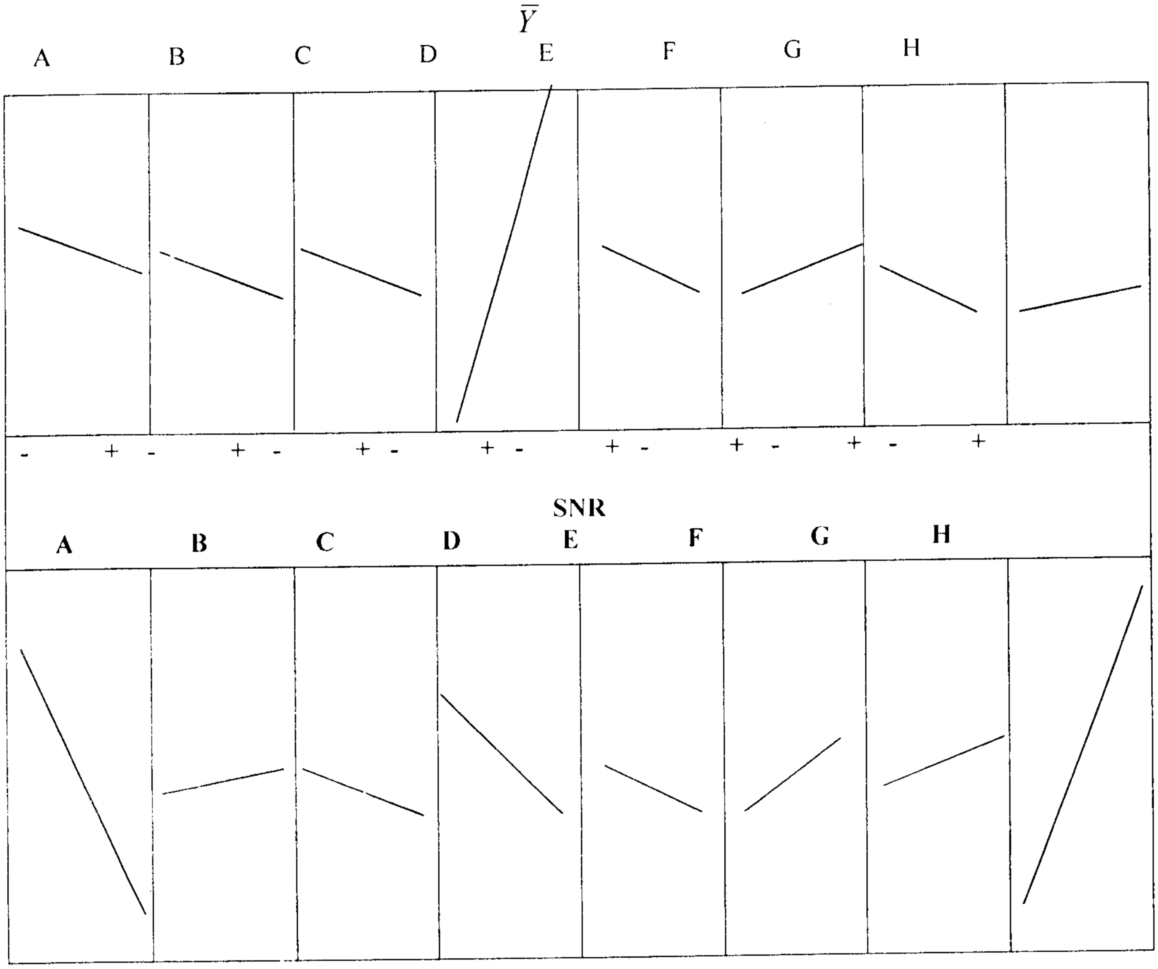

Figure 2: The plot main effects from Sensitivity ($\overline{Y}$) and SNR

The second example is a Dynamic Parameter Design. In this example there are 8 control factors, each at 2-levels. The control array is a 16-run of 2^{8-4} fractional factorial design. The layout is shown in Table 2, where the outer array consists of 3 levels of signal factor (M_1, M_2 & M_3) crossed with 2 levels of noise factor for a total of 6 runs as replicates. The response Y_{ijk} corresponds to the ith setting of control factors, with j level of signal factor and k level of noise factor. For each ith setting, the estimated slopes, $\hat{\beta}_i$ and standard deviation, $\hat{\sigma}_i$ are obtained using the following model:

$$y_{ijk} = \beta_i M_j + e_{ijk}$$

The model is through the origin and M_j are the signal factor. $j = 1,2,3$ and $i = 1,2,3,\cdots,16$, the $\hat{\sigma}$ is estimated unbiasedly by the Mean Square Error (MSE). Both $\hat{\beta}_i$ and $\hat{\sigma}_i$ are used in obtaining the Taguchi SNR and these form the last three columns of Table 2.

Table 2: Control Array of Dynamic Design with estimated Sensitivity (β) and SNR

Control Array								M_1		M_2		M_3				
A	B	C	D	E	F	G	H	N_1	N_2	N_1	N_2	N_1	N_2	$\hat{\beta}$	$\hat{\sigma}$	SNR
-	-	-	-	-	-	-	-	119.24	123.85	239.96	244.48	359.85	356.48	1.21	2.74	-7.10
+	-	-	-	-	+	+	+	155.68	164.28	314.20	322.74	471.62	482.16	1.59	5.12	-10.12
-	+	-	-	+	-	+	+	129.01	136.18	261.76	267.64	392.96	400.10	1.32	3.71	-8.96
+	+	-	-	+	+	-	-	168.23	176.22	339.77	348.08	510.13	519.96	1.72	4.81	-8.95
-	-	+	-	+	+	+	-	142.58	150.82	289.41	297.00	434.55	444.01	1.46	4.64	-10.02
+	-	+	-	+	-	-	+	160.04	168.76	323.79	331.25	486.42	496.04	1.64	4.75	-9.26
-	+	+	-	-	+	-	+	142.67	149.85	288.54	294.83	433.05	441.05	1.46	3.95	- 8.67
+	+	+	-	-	-	+	-	151.63	160.31	307.04	314.60	460.32	470.47	1.55	4.88	-9.95
-	-	-	+	+	+	-	+	165.32	174.81	335.62	343.71	503.86	514.25	1.70	5.14	-9.62
+	-	-	+	+	-	+	-	186.01	199.49	378.22	390.93	568.48	584.55	1.92	7.76	-12.12
-	+	-	+	-	+	+	-	141.85	149.26	287.65	294.23	430.93	439.74	1.45	4.22	-9.26
+	+	-	+	-	-	-	+	184.97	195.47	373.78	383.86	561.27	574.13	1.89	6.17	-10.26
-	-	+	+	-	-	+	+	148.68	157.58	302.37	310.42	453.34	463.59	1.53	5.01	-10.30
+	-	+	+	-	+	-	-	180.66	191.01	365.93	375.84	549.52	562.05	1.85	6.03	-10.25
-	+	+	+	+	-	-	-	154.99	163.52	314.39	322.05	471.88	480.71	1.59	4.60	-9.23
+	+	+	+	+	+	+	+	182.84	196.52	371.72	384.37	558.93	574.23	1.89	7.64	-12.13

The plots of main effect from Sensitivity and SNR are shown in Figure 3 and a close examination revealed that factors A and D are significant to the sensitivity while factors A, D and G are significant to the dispersion (SNR). The half-normal plots and multiple regressions revealed the same result. The half-normal plots for both sensitivity and SNR are shown in figure 4(a) and 4(b) respectively. This shows one of the limitations of Taguchi's SNR, here A and D are significant to both sensitivity and dispersion effects. Because of this cross talk between sensitivity and dispersion effects, the SNR analysis misidentified factors A and D as having important dispersion effects.

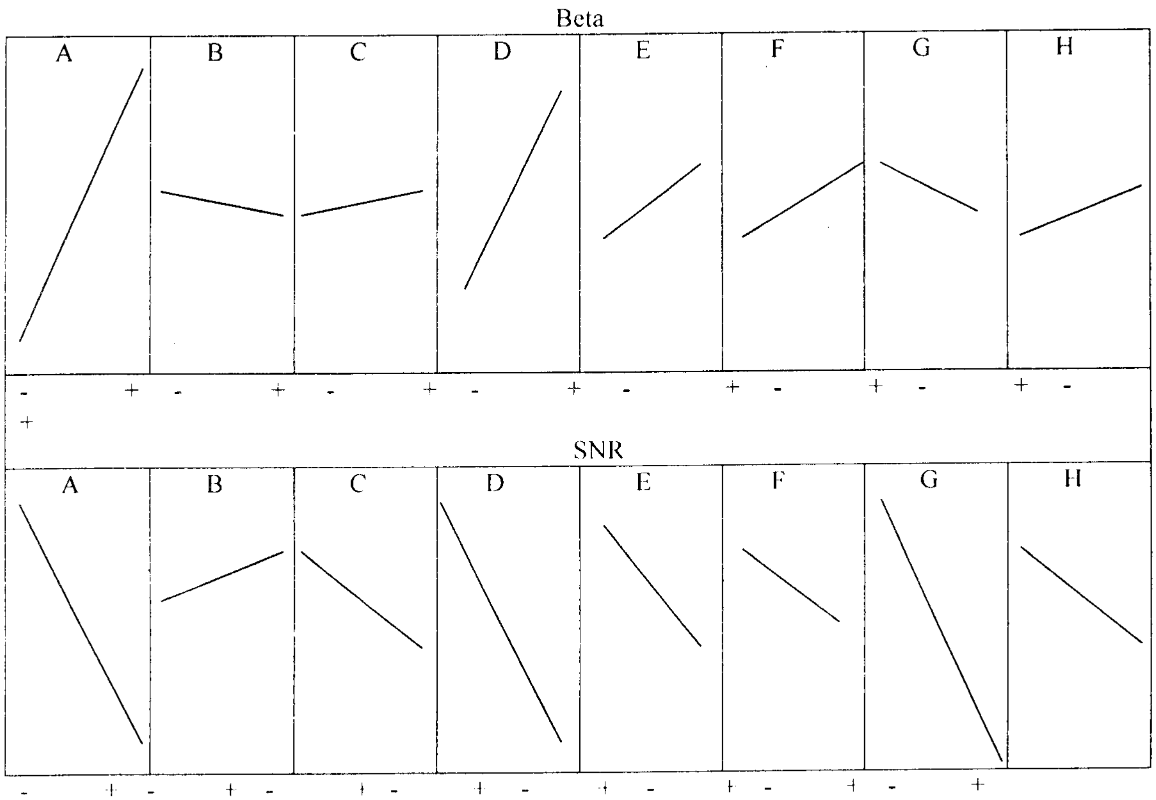

Figure 3: The plot main effects from Sensitivity (β) and SNR

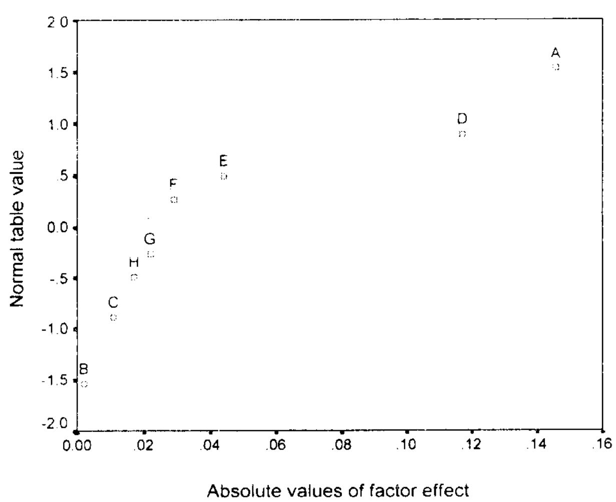

Fig. 4(b) Half-Normal plot for SNR

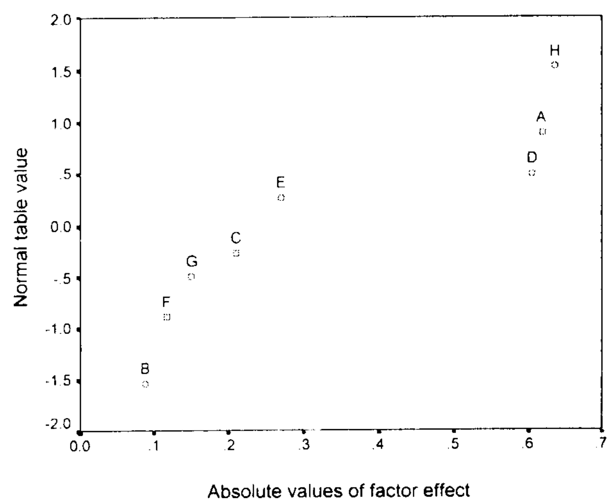

Fig. 4(a) Half-Normal plot for Beta (β)

Looking back at Taguchi SNR i.e

$$SNR = 10Log\left(\frac{\hat{\beta}_i^2}{\hat{\sigma}_i^2}\right) \text{.........................4(a), which can be re-written as}$$

$$SNR = 10Log\left(\frac{\hat{\beta}_i^\gamma}{\hat{\sigma}_i^2}\right) \text{.........................4(b)}$$

The problem could have been the wrong choice of γ which can result into the problem of cross-talk. Therefore it is appropriate to determine the value of γ that will give a clear cut between factors affecting sensitivity and those affecting dispersion.

This is achieved by varying the value of γ in the Taguchi's SNR equation (equation. 4b), and for each value of γ (γ ranges from -5 to 5), multiple regression of SNR on all the factors is obtained and the t-statistic(s) are computed to determine which of the factor(s) are significant. The computed t-statistic are then plotted against the values of γ to obtain the gamma plot (γ-plot). The table of t-statistic of the factors at various values of γ is shown in Table 3.

Table 3: Table of computed t-statistics at various values of γ

				Computed t-statistics				
Gamma	A	B	C	D	E	F	G	H
-5.0	-6.91	0.20	-1.06	-5.80	-2.31	-1.54	-0.43	-1.04
-4.5	-6.88	0.21	-1.08	-5.80	-2.31	-1.53	-0.51	-1.05
-4.0	-6.86	0.22	-1.09	-5.79	-2.31	-1.52	-0.61	-1.05
-3.5	-6.82	0.24	-1.11	-5.79	-2.31	-1.51	-0.72	-1.06
-3.0	-6.79	0.25	-1.13	-5.79	-2.32	-1.50	-0.84	-1.07
-2.5	-6.74	0.27	-1.15	-5.78	-2.32	-1.48	-0.98	-1.08
-2.0	-6.69	0.29	-1.18	-5.77	-2.32	-1.46	-1.15	-1.08
-1.5	-6.63	0.31	-1.21	-5.76	-2.32	-1.44	-1.35	-1.09
-1.0	-6.55	0.34	-1.24	-5.76	-2.32	-1.41	-1.58	-1.11
0.0	-6.31	0.41	-1.34	-5.67	-2.31	-1.34	-2.20	-1.13
1.0	-5.89	0.52	-1.47	-5.52	-2.28	-1.20	-3.18	-1.17
1.5	-5.52	0.60	-1.57	-5.36	-2.24	-1.09	-3.90	-1.19
2.0	-4.96	0.71	-1.68	-5.09	-2.16	-0.93	-4.86	-1.20
2.5	-4.06	0.85	-1.81	-4.61	-2.00	-0.68	-6.13	-1.20
3.0	-2.59	1.01	-1.92	-3.71	-1.69	-0.28	-7.72	-1.14
3.5	-0.32	1.16	-1.91	-2.15	-1.12	0.29	-9.28	-0.97
4.0	2.42	1.18	-1.66	-0.02	-0.29	0.94	-9.95	-0.64
4.5	4.66	1.05	-1.22	1.96	0.51	1.42	-9.33	-0.25
5.0	5.99	0.88	-0.78	3.30	1.08	1.66	-8.12	0.06

The resulting γ-plot is shown in Figure 5. From the plot, when $\gamma = 2$, which is equivalent to Taguchi's SNR, factor A, D and G are significant, i.e distinctly different from the remaining 5 factors. But when $\gamma = 4$, the other two factors, factors A and D have joined the remaining 5 factors leaving only G as the only distinct factor affecting the dispersion effect. Therefore at an appropriate choice of γ (when $\gamma = 4$), only factor G is significant to dispersion effect and factors A and D are significant to the mean effect.

Taking $\gamma = 4$ and using it to obtain the $SNR = 10Log\left(\frac{\hat{\beta}_i^4}{\hat{\sigma}_i^2}\right)$.

The regression analysis shows that only factor G is significant to SNR, which gives

$\hat{\eta} = -5.698 - 0.719x_G$ and sensitivity model gives

$\hat{\beta} = 1.611 + 0.146x_A + 0.117x_D$

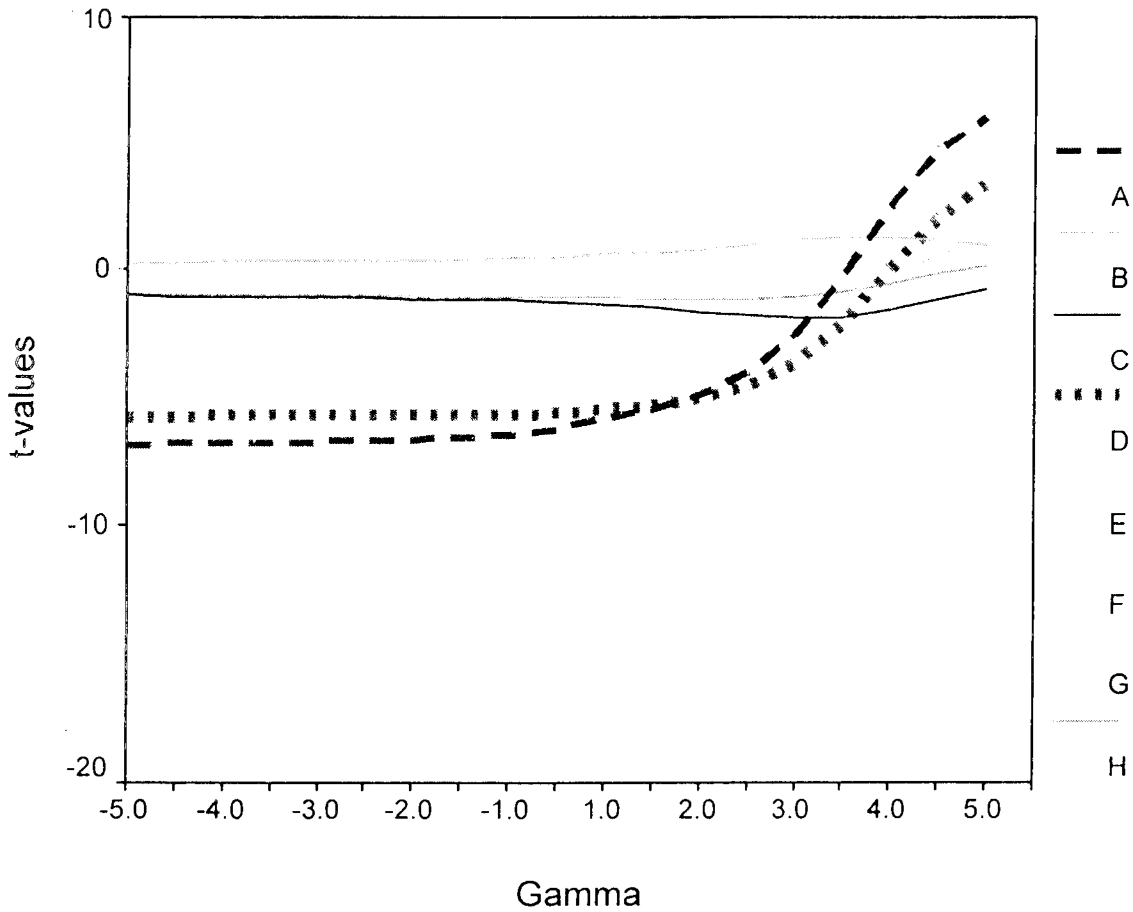

Figure 5: Plot of t-values against values of γ (Gamma plot).

CONCLUSION

The results show that Taguchi's SNR as measure of performance is often appropriate when dealing with Static Parameter Design as there is always no problem of cross talk among the factors because of distinction between factors affecting sensitivity (Target) and those affecting dispersion (variability).In Static Parameter design example, there is distinction between factors affecting the mean and those affecting the variability; only factor D was significant to the mean effect while factors A and H were significant to the dispersion effect.

Dealing with Dynamic Parameter Design requires estimation of γ, it should always be assumed unknown as indicated from the second example. A wrong choice of γ by Taguchi's approach resulted into the cross talk between factors affecting sensitivity and those affecting dispersion. Factors A and D were significant to the mean effect and the same factors A and D together with G were significant to dispersion effect, showing that factors A and D are significant to both the mean and dispersion effects. But a right choice of γ ($\gamma = 4$) eliminated the problem.

This suggests that when dealing with Dynamic or Static Parameter Design the γ in the functional relationship between sensitivity and variation should always assumed unknown and be estimated through gamma plot.

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