

BAYESIAN PANEL DATA PARADIGM ON DETERMINANT OF WEST AFRICAN ECONOMIC GROWTH

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Abstract

This study investigates impacts of the growth determinants of West African economy via Bayesian mixed panel data estimator. The choice of Bayesian estimator is not by accident since it has been established in the literature that it performed better than the frequentist. Estimates were obtained calculating the location and spread of Markov Chain Monte Carlo of Bayesian panel data. The study found out that Capital services at constant 2011 national prices, Welfare-relevant TFP at constant national prices, Share of labour compensation in GDP at current national prices and Real internal rate of return contribute negatively towards economy of west Africa countries during the period of study, this implies that governments of west African countries need to put all hands on deck to improve the economy of the region. They to harness resources and reach common goal for speed developments. Thus, TFP at constant national prices, Average depreciation rate of the capital stock, Real consumption at constant 2011 national prices, Real domestic absorption at constant 2011 national prices and Capital stock at constant 2011 national prices contributed minimally and positively towards economy of west African countries during the 17years period of study. The therefore recommend that there should be total overhauling of the region.

Keywords: Markov Chain Monte Carlo Method, Heteroscedasticity, Gibbs Sampler, Bayesian Inference.

AMS Subj. Classification: 62J05, 62J12, 62F15, 65C05

1.0 Introduction

Ulrich (2005) pointed out that prior information can be systematically be included in the analysis when Bayesian technique is adopted in the study. This superimposes the use of frequentist approach to making the inferences about the parameter. Oloyede *et al* (2013 & 2014) observed that Bayesian approach in the making of inferences particularly when correct priors choosing to improve the performances of the estimators and lead to valid inferences about the parameter even in the event of heteroscedasticity. Vanden *et al* (1994) were first to suggest the integration of Bayesian approach to panel data where the approach was examined via composed error model. Koop *et al* (1997) adopted Bayesian approach to examine both fixed and random effects model. Tsiomas (2006) worked on a dynamic stochastic frontier model with the application of Bayesian method. Sickles *et al* (2014) proposed a Bayesian technique in a panel data model with effect specifying a factor model setting. Ley and Mark (2009) examined model uncertainty which is associated with variable selection via linear regression models where cross-country growth regression was the focus. Durlauf (2001), Brock *et al* (2003) examined model uncertainty in growth regression. The work of Chipman *et*

al (2001), Ley and Mark (2009) examined in detail Bayesian model averaging with reference to panel data.

It has been established from the literature that many economic relationships are dynamic in nature. Balestra and Nerlove (1966) examined dynamic demand for natural gas, Baltagi and Levin (1986) worked on dynamic demand for additive commodity, Arellano and Bond (1991) looked into dynamic model on employment, Blundel *et al* (1992) examined dynamic model on company investment, Islam (1995) worked on dynamic model for growth convergence.

Roberto and Thanabalasingam (2013) examined the determinants of growth in the Asian economics where they found out that the economy's ratio and trade openness are positively related and contributed to the growth but government consumption are inversely related to the growth in Asian developing economy. Their approach was the Bayesian paradigm under dynamic panel data. They concluded that inflation impeded economic growth when it exceeds 5.43% in the Asian economy. Ulrich (2005) concluded that Bayesian panel data is appropriate when the available data do not provide enough information while enough extraneous information is available. Park *et al* (2007) examined semiparametric efficient panel data estimators under a variety of specifications which includes a misspecified dynamic model that lead to the presence of an idiosyncratic error of autoregressive pattern. Junrong (2016) examined panel data model with unobserved heterogeneous time-varying effects where the individual effect was treated as a random function of time and favorably compared with Kneip *et al* (2012) KSS estimator, they concluded that Bayesian estimator expressed consistent superior performance under various data generating processes, especially when compared with estimators that depend on a parametric assumption since their estimator did not rely on parametric assumption. They asserted that parametric type estimator allowed for temporal variation which has the tendency of having idiosyncratic misspecification of the temporal pattern of the individual effects, but they found out that Bayesian estimator without parametric assumption uniformly dominate the misspecification problems and have finite sample consistent performance over Hsiao, Pesaran, and Tahmiscioghi's estimator.

Enrique (2010) found out in a panel data setting, the price of investment goods, distance to major world cities and political rights are the robust determinant of economic growth. Huang *et al* (2011) extended Bayesian model averaging framework to panel data model and established that it was appropriate for inference in short dynamic panel data models with endogeneous regressors in the context of model uncertainty. Djula and Luka (2020) examining the Industrial Energy Consumption Determinants and adopted the approach of A Panel Bayesian Model Averaging.

The study adopted Bayesian mixed due to its probabilistic nature and being affirmed in the literature that it outperformed classical approach. Sequel to section which reviewed past work is the section that examined methodical nature of the study. Section three dealt with data analysis and interpretation. Conclusion is in section four. The study contains references and appendix.

2.0 Material and Methods

Considering a static panel model of the form $y_{it} = X_{it}\beta + z_{it}\gamma + u_{it}$ where z_{it} denotes the covariates of dummies variables constructed for the individual specific effect (countries) and temporal effect (time-variant), $i = 1, \dots, N$ $t = 1, \dots, T$; γ is a scalar which represents the

parameters of dummies variables, X_{it} is $1 \times k$ of explanatory variables which are determinant of growth while β is $k \times 1$ parameters of independent variables.

$$\text{It can be expressed as } y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} X_1 & j_T & 0 & \dots & 0 \\ X_2 & 0 & j_T & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_N & 0 & \dots & 0 & j_T \end{bmatrix} \begin{bmatrix} \beta \\ \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} + \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} \quad (1)$$

2.1 Likelihood function

Given the Bayesian framework and for a given model stated above with one component error, the study composed the following assumptions:

$$u_{it} | y_{it-1} \dots y_{i0}, x_i, z_i, u_{it} \sim N(0, \sigma_v^2) \quad (2)$$

$$v_{it} | y_{i0}, x_i^j, z_i^j \sim N(\theta \bar{y}_{i0} + \gamma \bar{x}_i, \sigma_u^2) \quad (3)$$

where $\bar{x}_i^j$ is the time series mean of the main variables X_i^j with temporal effect T and individual effect (N), the likelihood was derived with the consideration of the above error density function, based on the above static panel model under normality of the non-spherical disturbances, the likelihood function can be expressed as:

$$L(\alpha, \beta, \gamma | \phi^2, \sigma_v^2) = -\frac{NT}{2\sigma_v^2} \exp \left(-\frac{1}{2\sigma_v^2} (y_i - X_i\beta - z_{it}\gamma)' \Omega^{-1} (y_i - X_i\beta - z_{it}\gamma) \right) \quad (4)$$

where $\Omega = \sigma_v^2 \Sigma$, $\phi^2 = \sigma_v^2 / \sigma_i^2$ and $\Sigma = Q + \phi^{-2} P$ Baltagi(2005).

Prior

$$p(\beta, \gamma, \alpha, \sigma, \phi) = p(\beta, \gamma) p(\sigma) p(\alpha) p(\phi) \quad (5a)$$

Posterior density:

$$\begin{aligned} \pi(\beta, \gamma, \sigma, D | y) &\propto \sigma^{nT/2} \exp \left[-\sigma \frac{(y_{it} - X'_{it}\beta - z'_{it}\gamma)' \Omega^{-1} (y_{it} - X'_{it}\beta - z'_{it}\gamma)}{2} \right] \times \exp \left[-\frac{1}{2} (\beta - \beta_0)' B_0^{-1} (\beta - \beta_0) \right] \\ &\times \sigma^{\alpha_0/2} \exp \left[-\frac{\delta_0 \sigma}{2} \right] \times |D|^{-K_2/2} \exp \left[-\frac{1}{2} b' D^{-1} b \right] \times \\ &|D|^{-(v_0 - K_2 - 1)/2} \exp \left[-\frac{1}{2} \text{tr}(D_0^{-1} D^{-1}) \right] \quad (5b) \end{aligned}$$

where $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_n)$ depending on the number of dummies variables.

$$\sigma | y, \beta, b, D \sim G(\alpha_1 \setminus 2, \delta_1 \setminus 2) \quad (6)$$

where

$$\alpha_1 = \alpha_0 + nT \quad (7)$$

$$\delta_1 = \delta_0 + (y_{it} - X'_{it}\beta - z'_{it}\gamma)' (y_{it} - X'_{it}\beta - z'_{it}\gamma) \quad (8)$$

and

$$D^{-1} | \gamma \sim W_{K_2}(v_1, D_1) \quad (9a)$$

Where

$$v_1 = v_0 + K_2 \quad (9b)$$

$$D_1 = [D_1 + \gamma_i \gamma_i']^{-1} \quad (9c)$$

Maximizing 5a over, given ϕ^2 yields

$$\hat{\beta}_{mle} = [X'(Q + \phi^2(P - \bar{J}_{NT}))X]^{-1} X'[Q + \phi^2(P - \bar{J}_{NT})]y \quad (10)$$

$$\hat{\sigma}_{v.mle}^2 = d'[Q + \phi^2(P - J_{NT})]d/NT \quad (11)$$

$$\beta|y, \sigma, D \sim N_{K_1}(\bar{\beta}, B_1) \quad (12)$$

$$\text{Where } B_1 = [X'_{1i}B_{1i}^{-1}X_i + B_0^{-1}]^{-1} \quad (13)$$

$$\bar{\beta} = B_1[X'_{1i}B_{1i}^{-1}y_i + B_0^{-1}\beta_0] \quad (14)$$

Maximizing (5a) over ϕ^2 , given β yields

$$\hat{\phi}^2 = \frac{d'Qd}{(T-1)d'(P-J_{NT})d} \quad (15)$$

Stata16 StataCorp (2019) with Bayesian package was used to sample the marginal posteriors density of the panel data, the code is added to the appendix for reader consumption. Data was obtained from penworld table 9.1 with the variable names below:

2.2 National accounts-based variables

rgdpna	Real GDP at constant 2011 national prices (in mil. 2011US\$)
rconna	Real consumption at constant 2011 national prices (in mil. 2011US\$)
rdana	Real domestic absorption at constant 2011 national prices (in mil. 2011US\$)
rnna	Capital stock at constant 2011 national prices (in mil. 2011US\$)
rkna	Capital services at constant 2011 national prices (2011=1)
rtfpna	TFP at constant national prices (2011=1)
rwtfpna	Welfare-relevant TFP at constant national prices (2011=1)
labsh	Share of labour compensation in GDP at current national prices
irr	Real internal rate of return
delta	Average depreciation rate of the capital stock

3.0 Results

The study adopted Bayesian mixed modelling to calibrate west African national account variables which show case the status of economy of the sub-region of Africa.

Table 1: Showing pool panel data with no effects

	rgdpna	rkna	rtfpna	rwtfpna	labsh	irr	delta	rconna	rdana	rnna
rgdpna	1									
rkna	0.1115	1								
rtfpna	-0.1365	-0.4017	1							
rwtfpna	0.2369	-0.1489	0.6799	1						
labsh	-0.5541	0.0345	-0.1224	-0.3553	1					
irr	0.593	0.0257	0.016	0.1834	-0.6618	1				
delta	0.1286	0.3834	-0.0738	-0.1309	-0.2595	0.0848	1			
rconna	0.998	0.1103	-0.1641	0.2204	-0.5399	0.5886	0.119	1		
rdana	0.9972	0.1391	-0.1672	0.2435	-0.5485	0.5726	0.1248	0.9973	1	
rnna	0.9713	0.1583	-0.2157	0.216	-0.4879	0.5078	-0.0247	0.9708	0.9773	1

Table 1 above depicts the relationship amongst the national-account based variables "rgdna" is positively and highly correlated with "rconna", "rnna", and "rdana". Each of "rkna", "rtfpna", delta and "rwtfpna" is weakly related with all the variables in study.

Variables		Mean	Std. Dev.	Min	Max	Observations
rgdpna	overall	9.6433	1.4696	7.2284	13.8297	N = 272
	between		1.4910	7.6599	13.4765	n = 16
	within		0.2605	8.9888	10.2969	T = 17
rkna	overall	0.9291	0.3698	0.1568	2.5855	N = 187
	between		0.1127	0.6707	1.0616	n = 11
	within		0.3538	0.3003	2.4531	T = 17
rtfpna	overall	1.0445	0.1408	0.7884	1.7779	N = 153
	between		0.0978	0.9459	1.2831	n = 9
	within		0.1061	0.7614	1.5394	T = 17
rwtfpna	overall	1.0130	0.1444	0.6419	1.4306	N = 153
	between		0.1117	0.8350	1.2006	n = 9
	within		0.0985	0.6627	1.2937	T = 17
labsh	overall	0.5247	0.1299	0.2864	0.8062	N = 187
	between		0.1264	0.3809	0.8062	n = 11
	within		0.0476	0.3945	0.6619	T = 17
irr	overall	0.1097	0.0832	0.01	0.3075	N = 187
	between		0.0773	0.01	0.2513	n = 11
	within		0.0383	-0.0552	0.2352	T = 17
delta	overall	0.0473	0.0152	0.0175	0.0921	N = 272
	between		0.0139	0.0203	0.0775	n = 16
	within		0.0070	0.0275	0.0731	T = 17
rconna	overall	9.5339	1.4289	6.9201	13.5439	N = 272
	between		1.4484	7.6762	13.2889	n = 16
	within		0.2608	8.5728	10.2814	T = 17
rdana	overall	9.7264	1.4302	7.1551	13.7196	N = 272
	between		1.4452	7.7248	13.4182	n = 16
	within		0.2829	8.7767	10.4473	T = 17
rnna	overall	10.7056	1.4928	8.5149	14.6091	N = 272
	between		1.5146	8.5774	14.2517	n = 16
	within		0.2643	10.0419	11.4259	T = 17

Table 2 described descriptive statistics of the economics variables for overall, between and within. The mean, standard deviation, minimum and maximum of each of the variables was examined.

Table 3: showing temporal effects of Growth determinants of West Africa Economy between 2001-2017

Bayesian multilevel regression			MCMC iterations	=	12,500
Metropolis-Hastings and Gibbs sampling			Burn-in	=	2,500
			MCMC sample size	=	10,000
Group variable: countries			Number of groups	=	9
			Obs per group:		
			min	=	17
			avg	=	17
			max	=	17
			Number of obs	=	153
			Acceptance rate	=	0.8395
			Efficiency: min	=	0.0015
			avg	=	0.0626
			max	=	0.4839
Log marginal-likelihood					
rgdpna	Mean	Std. Dev.	MCSE	Median	[95% Cred.Interval]
lrkna	-0.0168	0.0123	0.0029	-0.0172	-0.0405 0.0081
lrtpna	0.9969	0.0262	0.0012	0.9968	0.9457 1.0499
lrwtpna	-1.0211	0.0307	0.0033	-1.0220	-1.0812 -0.9591
llabsh	-0.0005	0.0110	0.0005	-0.0005	-0.0218 0.0212
lirr	-0.0010	0.0031	0.0002	-0.0011	-0.0071 0.0051
ldelta	0.0027	0.0143	0.0008	0.0028	-0.0258 0.0315
rconna	0.0026	0.0267	0.0011	0.0026	-0.0487 0.0559
rdana	1.0191	0.0351	0.0035	1.0197	0.9475 1.0862
rnna	0.0023	0.0179	0.0019	0.0023	-0.0339 0.0373
_cons	-0.3484	0.1555	0.0403	-0.3552	-0.6318 -0.0265
countries U0: σ^2	0.0159	0.0106	0.0009	0.0132	0.0054 0.0433
e.rgdpna σ^2	0.0002	0.00002	3.30E-07	0.0002	0.0001 0.0002

Table 3 described the output of Bayesian mixed modelling of growth determinants of West African economy. The number of iteration is 12500 with 2500 burn-in in an attempt to discard the effect of initial values, thus mcmc sample retained after burn-in is 10000, the data comprises of 9 group which is no of years, with 17 countries in West Africa. The study reported acceptance rate of 83%, with efficiency of 48%.

The economic variables "rkna"[Capital services at constant 2011 national prices (2011=1)], "rwtfpna"[Welfare-relevant TFP at constant national prices (2011=1)], "labsh"[Share of labour compensation in GDP at current national prices] and "irr"[Real internal rate of return] contribute negatively towards economy of west Africa countries during the period of study with the magnitudes of -0.0168, -1.0211, -0.0005 and -0.0001 respectively. Thus "rtfpna"[TFP at constant national prices (2011=1)], "ldelta"[Average depreciation rate of the capital stock], "rconna"[Real consumption at constant 2011 national prices (in mil. 2011US\$)], "rdana"[Real domestic absorption at constant 2011 national prices (in mil. 2011US\$)] and "rnna"[Capital stock at constant 2011 national prices (in mil. 2011US\$)] contributed positively towards economy of west African countries during the 17years period of study.

Conclusion

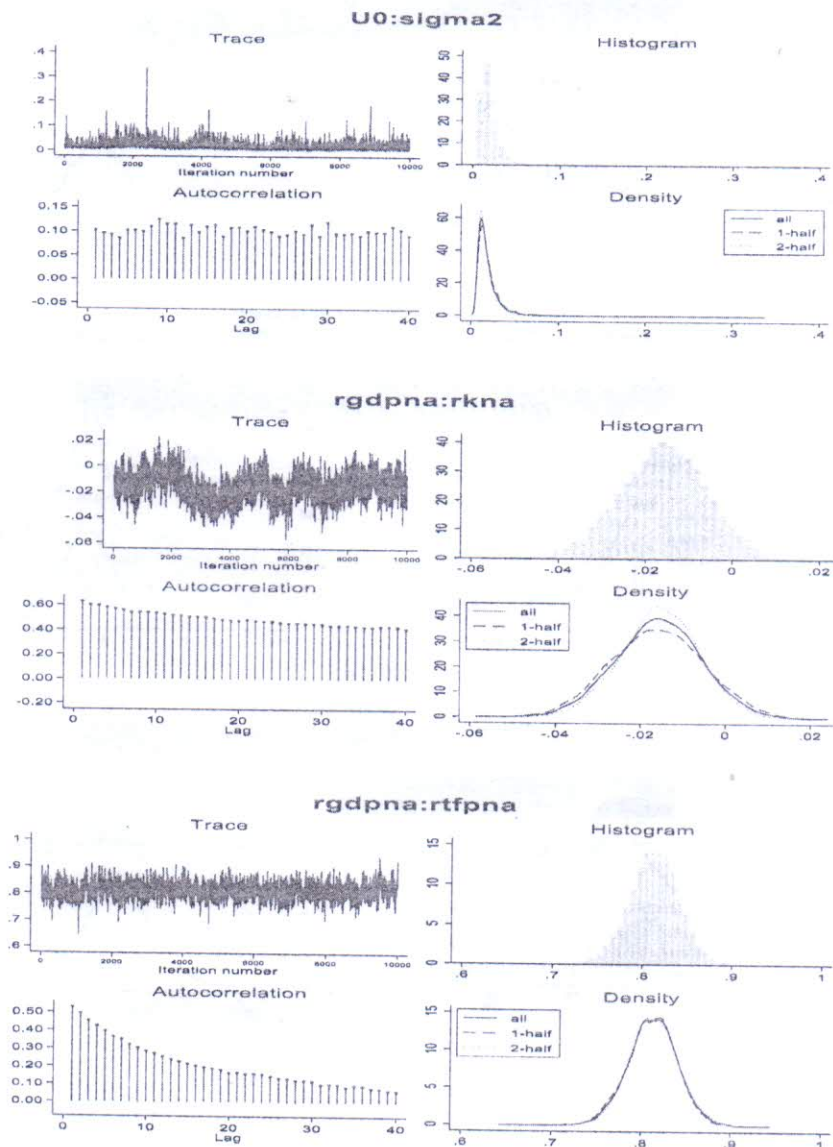
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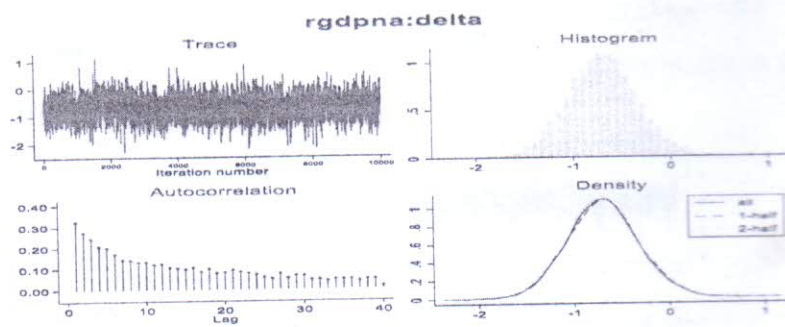
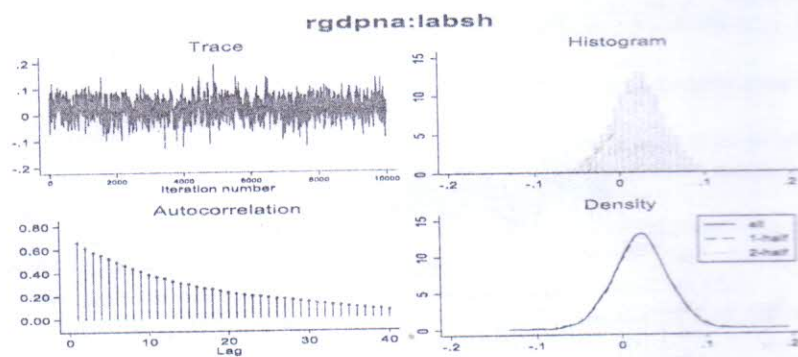
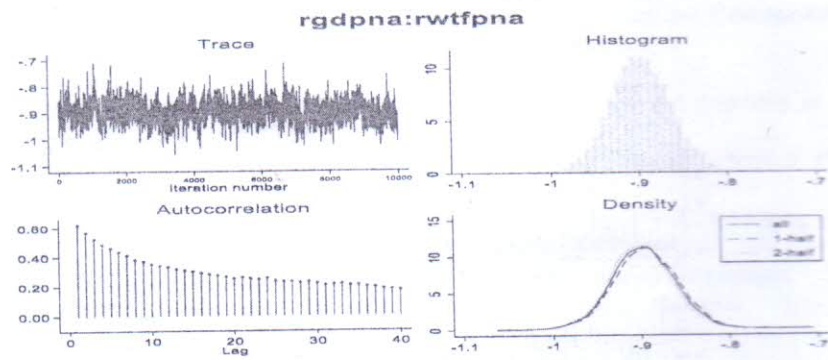
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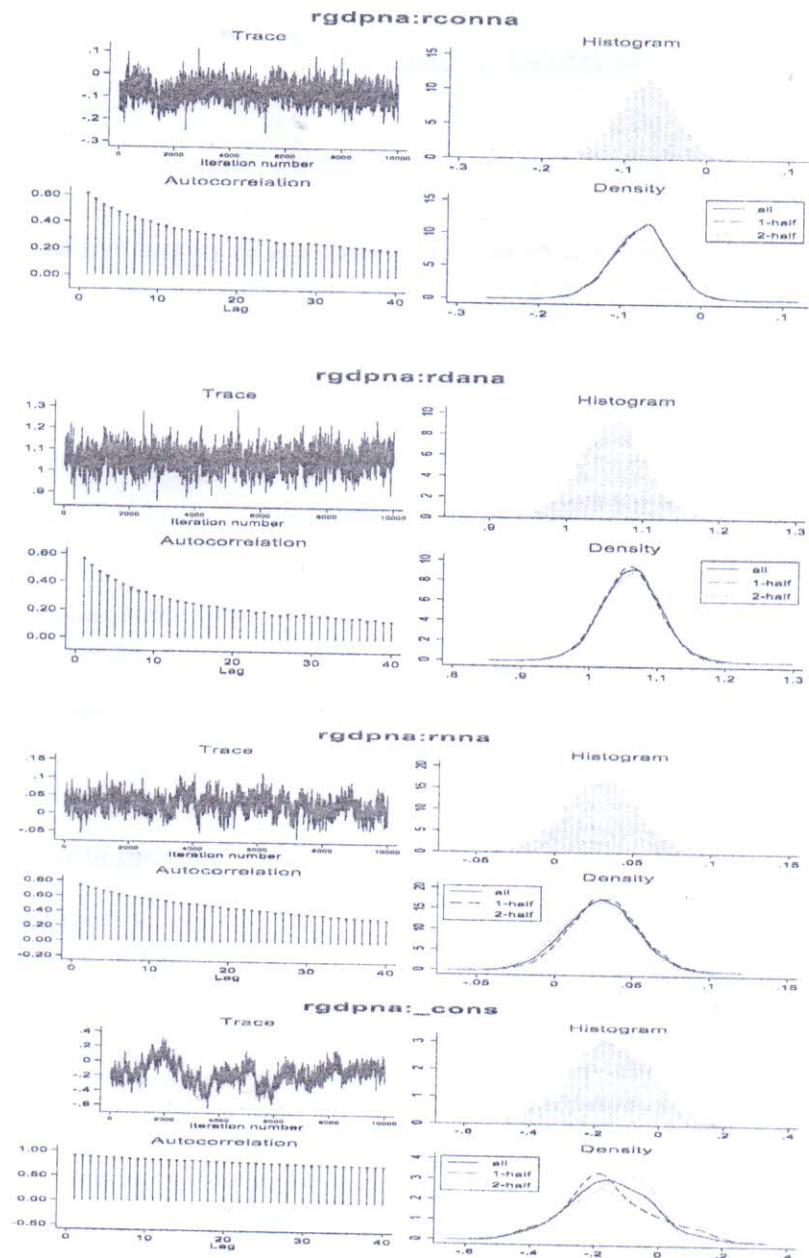
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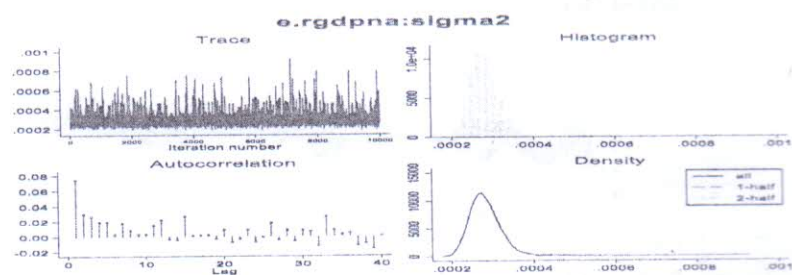
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Appendix









***All the charts in the appendix depict convergence of the parameters after 12500 iteration.